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GENERALIZED FRACTIONAL AND CIRCULAR TOTAL COLORINGS OF GRAPHS [1]

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Abstract

Let $\mathcal{P}$ and $\mathcal{Q}$ be additive and hereditary graph properties, $r, s \in \mathbb{N}$, $r \geq s$, and $[\mathbb{Z}_r]^s$ be the set of all s-element subsets of $\mathbb{Z}_r$. An (r, s)-*fractional*

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$(\mathcal{P}, \mathcal{Q})$-*total coloring* of G is an assignment $h : V(G) \cup E(G) \to [\mathbb{Z}_r]^s$ such that for each $i \in \mathbb{Z}_r$ the following holds: the vertices of G whose color sets contain color i induce a subgraph of G with property $\mathcal{P}$, edges with color sets containing color i induce a subgraph of G with property $\mathcal{Q}$, and the color sets of incident vertices and edges are disjoint. If each vertex and edge of G is colored with a set of s consecutive elements of $\mathbb{Z}_r$ we obtain an (r, s)-*circular* $(\mathcal{P}, \mathcal{Q})$-*total coloring* of G. In this paper we present basic results on (r, s)-fractional/circular $(\mathcal{P}, \mathcal{Q})$-total colorings. We introduce the fractional and circular $(\mathcal{P}, \mathcal{Q})$-total chromatic number of a graph and we determine this number for complete graphs and some classes of additive and hereditary properties.

Keywords: graph property, $(\mathcal{P}, \mathcal{Q})$-total coloring, circular coloring, fractional coloring, fractional $(\mathcal{P}, \mathcal{Q})$-total chromatic number, circular $(\mathcal{P}, \mathcal{Q})$-total chromatic number.

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References

[1] I. Bárány, *A short proof of Kneser's Conjecture*, J. Combin. Theory Ser. A **25** (1978) 325–326.
doi:10.1016/0097-3165(78)90023-7

[2] M. Borowiecki, A. Kemnitz, M. Marangio and P. Mihók, *Generalized total colorings of graphs*, Discuss. Math. Graph Theory **31** (2011) 209–222.
doi:10.7151/dmgt.1540

[3] M. Borowiecki and P. Mihók, *Hereditary properties of graphs*, in: V.R. Kulli, Ed., Advances in Graph Theory, Vishwa International Publications, Gulbarga (1991) 42–69.

[4] M. Borowiecki, I. Broere, M. Frick, P. Mihók and G. Semanišin, *Survey of hereditary properties of graphs*, Discuss. Math. Graph Theory **17** (1997) 5–50.
doi:10.7151/dmgt.1037

[5] F.R.K. Chung, *On the Ramsey numbers* $N(3, 3, \ldots, 3; 2)$, Discrete Math. **5** (1973) 317–321.
doi:10.1016/0012-365X(73)90125-8

[6] M.J. Dorfling and S. Dorfling, *Generalized edge-chromatic numbers and additive hereditary properties of graphs*, Discuss. Math. Graph Theory **22** (2002) 349–359.
doi:10.7151/dmgt.1180

[7] A. Hackmann and A. Kemnitz, *Circular total colorings of graphs*, Congr. Numer. **158** (2002) 43–50.

[8] R.P. Jones, *Hereditary properties and* $\mathcal{P}$*-chromatic numbers*, in: Combinatorics, London Math. Soc. Lecture Note, (Cambridge Univ. Press, London) **13** (1974) 83–88.

[9] G. Karafová, *Generalized fractional total coloring of complete graphs*, Discuss. Math. Graph Theory **33** (2013) 665–676.
doi:10.7151/dmgt.1697

[10] K. Kilakos and B. Reed, *Fractionally colouring total graphs*, Combinatorica **13** (1993) 435–440.
doi:10.1007/BF01303515

[11] L. Lovász, *Kneser's conjecture, chromatic number, and homotopy*, J. Combin. Theory Ser. A **25** (1978) 319–324.
doi:10.1016/0097-3165(78)90022-5

[12] P. Mihók, Zs. Tuza and M. Voigt, *Fractional $\mathcal{P}$-colourings and $\mathcal{P}$-choice-ratio*, Tatra Mt. Math. Publ. **18** (1999) 69–77.

[13] E.R. Scheinerman and D.H. Ullman, Fractional Graph Theory (John Wiley & Sons, New York, 1997).
http://www.ams.jhu.edu/$\sim$ers/fgt.

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