Discussiones Mathematicae
Graph Theory 35 (2015) 517–532
doi:10.7151/dmgt.1812

GENERALIZED FRACTIONAL AND CIRCULAR TOTAL COLORINGS OF GRAPHS [1]

Arnfried Kemnitz, Massimiliano Marangio

Computational Mathematics
Technical University Braunschweig, Germany

e-mail: a.kemnitz@tu-bs.de, m.marangio@tu-bs.de

Peter Mihók

Mathematical Institute, Slovak Academy of Sciences
Grešákova 6, 040 01 Košice, Slovak Republic
and
Department of Applied Mathematics and Business Informatics
Faculty of Economics, Technical University
B.Němcovej 32, 040 01 Košice, Slovak Republic

Janka Oravcová

Department of Applied Mathematics and Business Informatics
Faculty of Economics, Technical University
B.Němcovej 32, 040 01 Košice, Slovak Republic

e-mail: janka.oravcova@tuke.sk

and

Roman Soták

Institute of Mathematics
Faculty of Science, P.J. Šafárik University
Jesenná 5, 041 54 Košice, Slovak Republic

e-mail: roman.sotak@upjs.sk

Abstract

Let $\mathcal{P}$ and $\mathcal{Q}$ be additive and hereditary graph properties, $r, s \in \mathbb{N}$, $r \geq s$, and $[\mathbb{Z}_r]^s$ be the set of all s-element subsets of $\mathbb{Z}_r$. An (r, s)-*fractional*

[1]This work was supported by the Slovak Science and Technology Assistance Agency under the contract No. APVV-0023-10 and by the Slovak VEGA Grant 1/0652/12.

$(\mathcal{P}, \mathcal{Q})$-*total coloring* of G is an assignment $h : V(G) \cup E(G) \rightarrow [\mathbb{Z}_r]^s$ such that for each $i \in \mathbb{Z}_r$ the following holds: the vertices of G whose color sets contain color i induce a subgraph of G with property $\mathcal{P}$, edges with color sets containing color i induce a subgraph of G with property $\mathcal{Q}$, and the color sets of incident vertices and edges are disjoint. If each vertex and edge of G is colored with a set of s consecutive elements of $\mathbb{Z}_r$ we obtain an (r, s)-*circular* $(\mathcal{P}, \mathcal{Q})$-*total coloring* of G. In this paper we present basic results on (r, s)-fractional/circular $(\mathcal{P}, \mathcal{Q})$-total colorings. We introduce the fractional and circular $(\mathcal{P}, \mathcal{Q})$-total chromatic number of a graph and we determine this number for complete graphs and some classes of additive and hereditary properties.

Keywords: graph property, $(\mathcal{P}, \mathcal{Q})$-total coloring, circular coloring, fractional coloring, fractional $(\mathcal{P}, \mathcal{Q})$-total chromatic number, circular $(\mathcal{P}, \mathcal{Q})$-total chromatic number.

2010 Mathematics Subject Classification: 05C15, 05C75.

References

[1] I. Bárány, *A short proof of Kneser's Conjecture*, J. Combin. Theory Ser. A **25** (1978) 325–326.
doi:10.1016/0097-3165(78)90023-7

[2] M. Borowiecki, A. Kemnitz, M. Marangio and P. Mihók, *Generalized total colorings of graphs*, Discuss. Math. Graph Theory **31** (2011) 209–222.
doi:10.7151/dmgt.1540

[3] M. Borowiecki and P. Mihók, *Hereditary properties of graphs*, in: V.R. Kulli, Ed., Advances in Graph Theory, Vishwa International Publications, Gulbarga (1991) 42–69.

[4] M. Borowiecki, I. Broere, M. Frick, P. Mihók and G. Semanišin, *Survey of hereditary properties of graphs*, Discuss. Math. Graph Theory **17** (1997) 5–50.
doi:10.7151/dmgt.1037

[5] F.R.K. Chung, *On the Ramsey numbers* $N(3, 3, \ldots, 3; 2)$, Discrete Math. **5** (1973) 317–321.
doi:10.1016/0012-365X(73)90125-8

[6] M.J. Dorfling and S. Dorfling, *Generalized edge-chromatic numbers and additive hereditary properties of graphs*, Discuss. Math. Graph Theory **22** (2002) 349–359.
doi:10.7151/dmgt.1180

[7] A. Hackmann and A. Kemnitz, *Circular total colorings of graphs*, Congr. Numer. **158** (2002) 43–50.

[8] R.P. Jones, *Hereditary properties and* $\mathcal{P}$*-chromatic numbers*, in: Combinatorics, London Math. Soc. Lecture Note, (Cambridge Univ. Press, London) **13** (1974) 83–88.

[9] G. Karafová, *Generalized fractional total coloring of complete graphs*, Discuss. Math. Graph Theory **33** (2013) 665–676.
doi:10.7151/dmgt.1697

[10] K. Kilakos and B. Reed, *Fractionally colouring total graphs*, Combinatorica **13** (1993) 435–440.
doi:10.1007/BF01303515

[11] L. Lovász, *Kneser's conjecture, chromatic number, and homotopy*, J. Combin. Theory Ser. A **25** (1978) 319–324.
doi:10.1016/0097-3165(78)90022-5

[12] P. Mihók, Zs. Tuza and M. Voigt, *Fractional $\mathcal{P}$-colourings and $\mathcal{P}$-choice-ratio*, Tatra Mt. Math. Publ. **18** (1999) 69–77.

[13] E.R. Scheinerman and D.H. Ullman, Fractional Graph Theory (John Wiley & Sons, New York, 1997).
http://www.ams.jhu.edu/∼ers/fgt.

Received 8 May 2014
Revised 24 October 2014
Accepted 24 October 2014