

## EXTENDING THE MAX ALGORITHM FOR MAXIMUM INDEPENDENT SET

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### Abstract

The maximum independent set problem is an NP-hard problem. In this paper, we consider Algorithm MAX, which is a polynomial time algorithm for finding a maximal independent set in a graph  $G$ . We present a set of forbidden induced subgraphs such that Algorithm MAX always results in finding a maximum independent set of  $G$ . We also describe two modifications of Algorithm MAX and sets of forbidden induced subgraphs for the new algorithms.

**Keywords:** maximum independent set, stable set, stability number, independence number, reduction, graph transformation, MAX Algorithm, MIN Algorithm, Vertex Order Algorithm.

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