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GENERALIZED FRACTIONAL TOTAL COLORINGS OF GRAPHS

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Abstract

Let $\mathcal{P}$ and $\mathcal{Q}$ be additive and hereditary graph properties and let r, s be integers such that $r \geq s$. Then an $\frac{r}{s}$*-fractional* $(\mathcal{P}, \mathcal{Q})$*-total coloring* of a finite graph $G = (V, E)$ is a mapping f, which assigns an s-element subset of the set $\{1, 2, \ldots, r\}$ to each vertex and each edge, moreover, for any color i all vertices of color i induce a subgraph with property $\mathcal{P}$, all edges of color i induce a subgraph with property $\mathcal{Q}$ and vertices and incident edges have been assigned disjoint sets of colors. The minimum ratio of an $\frac{r}{s}$-fractional $(\mathcal{P}, \mathcal{Q})$-total coloring of G is called *fractional* $(\mathcal{P}, \mathcal{Q})$*-total chromatic number* $\chi''_{f,\mathcal{P},\mathcal{Q}}(G) = \frac{r}{s}$. We show in this paper that $\chi''_{f,\mathcal{P},\mathcal{Q}}$ of a graph G with $o(V(G))$ vertex orbits and $o(E(G))$ edge orbits can be found as a solution of a linear program with integer coefficients which consists only of $o(V(G)) + o(E(G))$ inequalities.

Keywords: fractional coloring, total coloring, automorphism group.

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