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BIPARTITION POLYNOMIALS, THE ISING MODEL AND DOMINATION IN GRAPHS

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Abstract

This paper introduces a trivariate graph polynomial that is a common generalization of the domination polynomial, the Ising polynomial, the matching polynomial, and the cut polynomial of a graph. This new graph polynomial, called the bipartition polynomial, permits a variety of interesting

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representations, for instance as a sum ranging over all spanning forests. As a consequence, the bipartition polynomial is a powerful tool for proving properties of other graph polynomials and graph invariants. We apply this approach to show that, analogously to the Tutte polynomial, the Ising polynomial introduced by Andrén and Markström in [3], can be represented as a sum over spanning forests.

Keywords: domination, Ising model, graph polynomial.

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