

COMPLETELY INDEPENDENT SPANNING TREES IN (PARTIAL) k -TREES

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Abstract

Two spanning trees T_1 and T_2 of a graph G are completely independent if, for any two vertices u and v , the paths from u to v in T_1 and T_2 are internally disjoint. For a graph G , we denote the maximum number of pairwise completely independent spanning trees by $\text{cist}(G)$. In this paper, we consider $\text{cist}(G)$ when G is a partial k -tree.

First we show that $\lceil k/2 \rceil \leq \text{cist}(G) \leq k - 1$ for any k -tree G . Then we show that for any $p \in \{\lceil k/2 \rceil, \dots, k - 1\}$, there exist infinitely many k -trees G such that $\text{cist}(G) = p$. Finally we consider algorithmic aspects for computing $\text{cist}(G)$. Using Courcelle's theorem, we show that there is a linear-time algorithm that computes $\text{cist}(G)$ for a partial k -tree, where k is a fixed constant.

Keywords: completely independent spanning trees, partial k -trees.

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