# ON MINIMAL GEODETIC DOMINATION IN GRAPHS 

Hearty M. Nuenay ${ }^{1}$<br>AND<br>Ferdinand P. Jamil ${ }^{2}$<br>Department of Mathematics and Statistics<br>MSU-Iligan Institute of Technology<br>Iligan City, Philippines<br>e-mail: ferdinand.jamil@g.msuiit.edu.ph


#### Abstract

Let $G$ be a connected graph. For two vertices $u$ and $v$ in $G$, a $u-v$ geodesic is any shortest path joining $u$ and $v$. The closed geodetic interval $I_{G}[u, v]$ consists of all vertices of $G$ lying on any $u-v$ geodesic. For $S \subseteq V(G), S$ is a geodetic set in $G$ if $\bigcup_{u, v \in S} I_{G}[u, v]=V(G)$.

Vertices $u$ and $v$ of $G$ are neighbors if $u$ and $v$ are adjacent. The closed neighborhood $N_{G}[v]$ of vertex $v$ consists of $v$ and all neighbors of $v$. For $S \subseteq V(G), S$ is a dominating set in $G$ if $\bigcup_{u \in S} N_{G}[u]=V(G)$. A geodetic dominating set in $G$ is any geodetic set in $G$ which is at the same time a dominating set in $G$. A geodetic dominating set in $G$ is a minimal geodetic dominating set if it does not have a proper subset which is itself a geodetic dominating set in $G$. The maximum cardinality of a minimal geodetic dominating set in $G$ is the upper geodetic domination number of $G$. This paper initiates the study of minimal geodetic dominating sets and upper geodetic domination numbers of connected graphs.


Keywords: minimal geodetic dominating set, upper geodetic domination number.
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    ${ }^{2}$ Corresponding author.

