

ON k -PATH PANCYCLIC GRAPHS

ZHENMING BI AND PING ZHANG

*Department of Mathematics
Western Michigan University
Kalamazoo, MI 49008, USA*

e-mail: ping.zhang@wmich.edu

Abstract

For integers k and n with $2 \leq k \leq n - 1$, a graph G of order n is k -path pancyclic if every path P of order k in G lies on a cycle of every length from $k + 1$ to n . Thus a 2-path pancyclic graph is edge-pancyclic. In this paper, we present sufficient conditions for graphs to be k -path pancyclic. For a graph G of order $n \geq 3$, we establish sharp lower bounds in terms of n and k for (a) the minimum degree of G , (b) the minimum degree-sum of nonadjacent vertices of G and (c) the size of G such that G is k -path pancyclic.

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