Discussiones Mathematicae Graph Theory 35 (2015) 215–227 doi:10.7151/dmgt.1793

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## ON •-LINE SIGNED GRAPHS $L_{\bullet}(S)$

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#### Abstract

A signed graph (or sigraph for short) is an ordered pair  $S = (S^u, \sigma)$ , where  $S^u$  is a graph, G = (V, E), called the underlying graph of S and  $\sigma$ :  $E \to \{+, -\}$  is a function from the edge set E of  $S^u$  into the set  $\{+, -\}$ . For a sigraph S its  $\bullet$ -line sigraph,  $L_{\bullet}(S)$  is the sigraph in which the edges of S are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in S have a vertex in common, any such L-edge ee'has the sign given by the product of the signs of the edges incident with the vertex in  $e \cap e'$ . In this paper we establish a structural characterization of  $\bullet$ -line sigraphs, extending a well known characterization of line graphs due to Harary. Further we study several standard properties of  $\bullet$ -line sigraphs, such as the balanced  $\bullet$ -line sigraphs, sign-compatible  $\bullet$ -line sigraphs and C-sign-compatible  $\bullet$ -line sigraphs.

**Keywords:** sigraph, line graph,  $\bullet$ -line sigraph, balance, sign-compatibility, C-sign-compatibility.

2010 Mathematics Subject Classification: 05C22, 05C75.

#### References

 B.D. Acharya, Signed intersection graphs, J. Discrete Math. Sci. Cryptogr. 13 (2010) 553-569. doi:10.1080/09720529.2010.10698314 M. Acharya and D. Sinha, Characterizations of line sigraphs, Nat. Acad. Sci. Lett. 28 (2005) 31-34.
 Extended abstract in Electron. Notes Discrete Math. 15 (2002) 12.

Extended abstract in: Electron. Notes Discrete Math. 15 (2003) 12.

- [3] M. Behzad and G.T. Chartrand, *Line coloring of signed graphs*, Elem. Math. 24(3) (1969) 49–52.
- [4] L.W. Beineke, Derived graphs and digraphs, in: Beiträge zur Graphentheorie, H. Sachs, H. Voss and H. Walter (Ed(s)), (Teubner, Leipzig, 1968) 17–33.
- [5] L.W. Beineke, Characterizations of derived graphs, J. Combin. Theory (B) 9 (1970) 129–135. doi:10.1016/S0021-9800(70)80019-9
- [6] M.K. Gill, Contribution to some topics in graph theory and its applications (Ph.D. Thesis, Indian Institute of Technology, Bombay, 1983).
- [7] F. Harary, On the notion of balance of a signed graph, Michigan Math. J. 2 (1953) 143-146. doi:10.1307/mmj/1028989917
- [8] F. Harary, Graph Theory (Addison-Wesley Publ. Comp., Reading, Massachusetts, 1969).
- F. Harary and R.Z. Norman, Some properties of line digraphs, Rend. Circ. Mat. Palermo (2) Suppl. 9 (1960) 161–168.
- [10] R.L. Hemminger and L.W. Beineke, *Line graphs and line digraphs*, in: Selected Topics in Graph Theory, L.W. Beineke and R.J. Wilson (Ed(s)), (Academic Press Inc., 1978) 271–305.
- [11] J. Krausz, Démonstration nouvelle d'une théorème de Whitney sur les réseaux, Mat. Fiz. Lapok 50 (1943) 75–89.
- [12] V.V. Menon, On repeated interchange graphs, Amer. Math. Monthly 73 (1966) 986– 989. doi:10.2307/2314503
- [13] O. Ore, Theory of Graphs (Amer. Math. Soc. Colloq. Publ. 38, Providence, 1962).
- [14] G. Sabidussi, Graph derivatives, Math. Z. 76 (1961) 385–401. doi:10.1007/BF01210984
- [15] E. Sampathkumar, *Point-signed and line-signed graphs*, Karnatak Univ. Graph Theory Res. Rep. No.1 (1973) (also see Abstract No. 1 in: Graph Theory Newsletter 2(2) (1972), National Academy Science Letters 7 (1984) 91–93).
- [16] D. Sinha, New frontiers in the theory of signed graph (Ph.D. Thesis, University of Delhi, Faculty of Technology, 2005).
- [17] D. Sinha and A. Dhama, Sign-compatibility of some derived signed graphs, Indian J. Math. 55 (2013) 23–40.
- [18] D. Sinha and A. Dhama, Canonical-sign-compatibility of some signed graphs, J. Combin. Inf. Syst. Sci. 38 (2013) 129–138.

- [19] D.B. West, Introduction to Graph Theory (Prentice-Hall of India Pvt. Ltd., 1996).
- H. Whitney, Congruent graphs and the connectivity of graphs, Amer. J. Math. 54 (1932) 150–168. doi:10.2307/2371086
- [21] T. Zaslavsky, A mathematical bibliography of signed and gain graphs and allied areas, 7th Edition, Electron. J. Combin. (1998) #DS8.
- [22] T. Zaslavsky, Glossary of signed and gain graphs and allied areas, Second Edition, Electron. J. Combin. (1998) #DS9.
- [23] T. Zaslavsky, Signed analogs of bipartite graphs, Discrete Math. 179 (1998) 205–216.
  doi:10.1016/S0012-365X(96)00386-X

Received 25 September 2013 Revised 7 April 2014 Accepted 9 April 2014