

## ON $\bullet$ -LINE SIGNED GRAPHS $L_\bullet(S)$

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### Abstract

A *signed graph* (or *sigraph* for short) is an ordered pair  $S = (S^u, \sigma)$ , where  $S^u$  is a graph,  $G = (V, E)$ , called the underlying graph of  $S$  and  $\sigma : E \rightarrow \{+, -\}$  is a function from the edge set  $E$  of  $S^u$  into the set  $\{+, -\}$ . For a sigraph  $S$  its  $\bullet$ -line sigraph,  $L_\bullet(S)$  is the sigraph in which the edges of  $S$  are represented as vertices, two of these vertices are defined adjacent whenever the corresponding edges in  $S$  have a vertex in common, any such  $L$ -edge  $ee'$  has the sign given by the product of the signs of the edges incident with the vertex in  $e \cap e'$ . In this paper we establish a structural characterization of  $\bullet$ -line sigraphs, extending a well known characterization of line graphs due to Harary. Further we study several standard properties of  $\bullet$ -line sigraphs, such as the balanced  $\bullet$ -line sigraphs, sign-compatible  $\bullet$ -line sigraphs and  $\mathcal{C}$ -sign-compatible  $\bullet$ -line sigraphs.

**Keywords:** sigraph, line graph,  $\bullet$ -line sigraph, balance, sign-compatibility,  $\mathcal{C}$ -sign-compatibility.

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