*Discussiones Mathematicae*
*Graph Theory 35 (2015) 121–132*
*doi:10.7151/dmgt.1785*



# MAXIMUM CYCLE PACKING IN EULERIAN GRAPHS USING LOCAL TRACES

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**Abstract**

For a graph $G = (V, E)$ and a vertex $v \in V$, let $T(v)$ be a *local trace* at $v$, i.e. $T(v)$ is an Eulerian subgraph of $G$ such that every walk $W(v)$, with start vertex $v$ can be extended to an Eulerian tour in $T(v)$.

We prove that every maximum edge-disjoint cycle packing $\mathcal{Z}^*$ of $G$ induces a maximum trace $T(v)$ at $v$ for every $v \in V$. Moreover, if $G$ is Eulerian then sufficient conditions are given that guarantee that the sets of cycles inducing maximum local traces of $G$ also induce a maximum cycle packing of $G$.

**Keywords:** edge-disjoint cycle packing, local traces, extremal problems in graph theory.

**2010 Mathematics Subject Classification:** 05C38.

## References

[1] S. Antonakopulos and L. Zhang, *Approximation algorithms for grooming in optical network design*, Theoret. Comput. Sci. **412** (2011) 3738–3751. doi:10.1016/j.tcs.2011.03.034

[2] F. Bäbler, *Über eine spezielle Klasse Euler'scher Graphen*, Comment. Math. Helv. **27** (1953) 81–100. doi:10.1007/BF02564555

[3] V. Bafna and P.A. Pevzner, *Genome rearrangement and sorting by reversals*, SIAM J. Comput. **25** (1996) 272–289. doi:10.1137/S0097539793250627

[4] A. Caprara, *Sorting permutations by reversals and Eulerian cycle decompositions*, SIAM J. Discrete Math. **12** (1999) 91–110.
doi:10.1137/S089548019731994X

[5] A. Caprara, A. Panconesi and R. Rizzi, *Packing cycles in undirected Graphs*, J. Algorithms **48** (2003) 239–256.
doi:10.1016/S0196-6774(03)00052-X

[6] G. Fertin, A. Labarre, I. Rusu, É. Tannier and S. Vialette, Combinatorics of Genome Rearrangement (MIT Press, Cambridge, Ma., 2009).

[7] J. Harant, D. Rautenbach, P. Recht and F. Regen, *Packing edge-disjoint cycles in graphs and the cyclomatic number*, Discrete Math. **310** (2010) 1456–1462.
doi:10.1016/j.disc.2009.07.017

[8] J. Harant, D. Rautenbach, P. Recht, I. Schiermeyer and E.-M. Sprengel, *Packing disjoint cycles over vertex cuts*, Discrete Math. **310** (2010) 1974–1978.
doi:10.1016/j.disc.2010.03.009

[9] J. Kececioglu and D. Sankoff, *Exact and approximation algorithms for sorting by reversals with application to genome rearrangement*, Algorithmica **13** (1997) 180–210.
doi:10.1007/BF01188586

[10] M. Krivelevich, Z. Nutov, M.R. Salvatpour, J. Yuster and R. Yuster, *Approximation algorithms and hardness results for cycle packing problems*, ACM Trans. Algorithm **3(4))** (2007) Article 48.

[11] O. Ore, *A problem regarding the tracing of graphs*, Elem. Math. **6** (1951) 49–53.

[12] P. Recht and E.-M. Sprengel, *Packing Euler graphs with traces*, in: Operations Research Proceedings, Klatte, Lüthi and Schmedders (Ed(s)), (Heidelberg, New York, Dordrecht, London, Springer, 2011) 53–58.

Received 29 December 2013
Accepted 10 March 2014