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ON UNIQUELY HAMILTONIAN CLAW-FREE AND TRIANGLE-FREE GRAPHS

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Abstract

A graph is uniquely Hamiltonian if it contains exactly one Hamiltonian cycle. In this note, we prove that claw-free graphs with minimum degree at least 3 are not uniquely Hamiltonian. We also show that this is best possible by exhibiting uniquely Hamiltonian claw-free graphs with minimum degree 2 and arbitrary maximum degree. Finally, we show that a construction due to Entringer and Swart can be modified to construct triangle-free uniquely Hamiltonian graphs with minimum degree 3.

Keywords: Hamiltonian cycle, uniquely Hamiltonian graphs, claw-free graphs, triangle-free graphs.

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