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GRAPHS WITH 3-RAINBOW INDEX $n-1$ AND $n-2$

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Abstract

Let $G = (V(G), E(G))$ be a nontrivial connected graph of order n with an edge-coloring $c : E(G) \to \{1, 2, \ldots, q\}$, $q \in \mathbb{N}$, where adjacent edges may be colored the same. A tree T in G is a *rainbow tree* if no two edges of T receive the same color. For a vertex set $S \subseteq V(G)$, a tree connecting S in G is called an S-tree. The minimum number of colors that are needed in an edge-coloring of G such that there is a rainbow S-tree for each k-subset S of $V(G)$ is called the k-rainbow index of G, denoted by $rx_k(G)$, where k is an integer such that $2 \leq k \leq n$. Chartrand *et al.* got that the k-rainbow index of a tree is $n-1$ and the k-rainbow index of a unicyclic graph is $n-1$ or $n-2$. So there is an intriguing problem: Characterize graphs with the k-rainbow index $n-1$ and $n-2$. In this paper, we focus on $k = 3$, and characterize the graphs whose 3-rainbow index is $n-1$ and $n-2$, respectively.

Keywords: rainbow S-tree, k-rainbow index.

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