# GRAPHS WITH 3-RAINBOW INDEX $n-1$ AND $n-2$ 

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#### Abstract

Let $G=(V(G), E(G))$ be a nontrivial connected graph of order $n$ with an edge-coloring $c: E(G) \rightarrow\{1,2, \ldots, q\}, q \in \mathbb{N}$, where adjacent edges may be colored the same. A tree $T$ in $G$ is a rainbow tree if no two edges of $T$ receive the same color. For a vertex set $S \subseteq V(G)$, a tree connecting $S$ in $G$ is called an $S$-tree. The minimum number of colors that are needed in an edge-coloring of $G$ such that there is a rainbow $S$-tree for each $k$-subset $S$ of $V(G)$ is called the $k$-rainbow index of $G$, denoted by $r x_{k}(G)$, where $k$ is an integer such that $2 \leq k \leq n$. Chartrand et al. got that the $k$-rainbow index of a tree is $n-1$ and the $k$-rainbow index of a unicyclic graph is $n-1$ or $n-2$. So there is an intriguing problem: Characterize graphs with the $k$-rainbow index $n-1$ and $n-2$. In this paper, we focus on $k=3$, and characterize the graphs whose 3 -rainbow index is $n-1$ and $n-2$, respectively.


Keywords: rainbow $S$-tree, $k$-rainbow index.
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