# THE 3-RAINBOW INDEX OF A GRAPH 

Lily Chen ${ }^{1}$, Xueliang Li ${ }^{1,2}$<br>Kang Yang ${ }^{1}$ and Yan Zhao ${ }^{1}$<br>Center for Combinatorics and LPMC-TJKLC<br>Nankai University<br>Tianjin 300071, China<br>e-mail: lily60612@126.com<br>lxl@nankai.edu.cn<br>yangkang@mail.nankai.edu.cn zhaoyan2010@mail.nankai.edu.cn


#### Abstract

Let $G$ be a nontrivial connected graph with an edge-coloring $c: E(G) \rightarrow$ $\{1,2, \ldots, q\}, q \in \mathbb{N}$, where adjacent edges may be colored the same. A tree $T$ in $G$ is a rainbow tree if no two edges of $T$ receive the same color. For a vertex subset $S \subseteq V(G)$, a tree that connects $S$ in $G$ is called an $S$-tree. The minimum number of colors that are needed in an edge-coloring of $G$ such that there is a rainbow $S$-tree for each $k$-subset $S$ of $V(G)$ is called the $k$-rainbow index of $G$, denoted by $r x_{k}(G)$. In this paper, we first determine the graphs of size $m$ whose 3 -rainbow index equals $m, m-1, m-2$ or 2 . We also obtain the exact values of $r x_{3}(G)$ when $G$ is a regular multipartite complete graph or a wheel. Finally, we give a sharp upper bound for $r x_{3}(G)$ when $G$ is 2-connected and 2-edge connected. Graphs $G$ for which $r x_{3}(G)$ attains this upper bound are determined.


Keywords: rainbow tree, $S$-tree, $k$-rainbow index.
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## References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory (GTM 244, Springer, 2008).
[2] Y. Caro, A. Lev, Y. Roditty, Zs. Tuza and R. Yuster, On rainbow connection, Electron. J. Combin. 15(1) (2008) R57.

[^0][3] G. Chartrand, G. Johns, K. McKeon and P. Zhang, Rainbow connection in graphs, Math. Bohem. 133 (2008) 85-98.
[4] G. Chartrand, F. Okamoto and P. Zhang, Rainbow trees in graphs and generalized connectivity, Networks 55 (2010) 360-367. doi:10.1002/net. 20339
[5] G. Chartrand, G. Johns, K. McKeon and P. Zhang, The rainbow connectivity of a graph, Networks 54(2) (2009) 75-81. doi:10.1002/net. 20296
[6] X. Li and Y. Sun, Rainbow Connections of Graphs (Springer Briefs in Math., Springer, 2012).
[7] X. Li, Y. Shi and Y. Sun, Rainbow connections of graphs: A survey, Graphs Combin. 29 (2013) 1-38. doi:10.1007/s00373-012-1243-2


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    ${ }^{2}$ Corresponding author.

