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THE 3-RAINBOW INDEX OF A GRAPH

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Abstract

Let G be a nontrivial connected graph with an edge-coloring $c : E(G) \to \{1, 2, \ldots, q\}$, $q \in \mathbb{N}$, where adjacent edges may be colored the same. A tree T in G is a rainbow tree if no two edges of T receive the same color. For a vertex subset $S \subseteq V(G)$, a tree that connects S in G is called an S-tree. The minimum number of colors that are needed in an edge-coloring of G such that there is a rainbow S-tree for each k-subset S of $V(G)$ is called the k-rainbow index of G, denoted by $rx_k(G)$. In this paper, we first determine the graphs of size m whose 3-rainbow index equals m, $m-1$, $m-2$ or 2. We also obtain the exact values of $rx_3(G)$ when G is a regular multipartite complete graph or a wheel. Finally, we give a sharp upper bound for $rx_3(G)$ when G is 2-connected and 2-edge connected. Graphs G for which $rx_3(G)$ attains this upper bound are determined.

Keywords: rainbow tree, S-tree, k-rainbow index.

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