

## ON DECOMPOSING REGULAR GRAPHS INTO ISOMORPHIC DOUBLE-STARS

SAAD I. EL-ZANATI, MARIE ERMETE

JAMES HASTY, MICHAEL J. PLANTHOLT

AND

SHAILESH TIPNIS

*Department of Mathematics*  
*Illinois State University*  
*Normal, Illinois 61790-4520, U.S.A.*

**e-mail:** saad@ilstu.edu  
ermet1mn@gmail.com  
HastyJ@bismarck.k12.il.us  
mikep@ilstu.edu  
tipnis@ilstu.edu

### Abstract

A *double-star* is a tree with exactly two vertices of degree greater than 1. If  $T$  is a double-star where the two vertices of degree greater than one have degrees  $k_1 + 1$  and  $k_2 + 1$ , then  $T$  is denoted by  $S_{k_1, k_2}$ . In this note, we show that every double-star with  $n$  edges decomposes every  $2n$ -regular graph. We also show that the double-star  $S_{k, k-1}$  decomposes every  $2k$ -regular graph that contains a perfect matching.

**Keywords:** graph decomposition, double-stars.

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