

NOTE

ON A SPANNING  $k$ -TREE IN WHICH SPECIFIED  
VERTICES HAVE DEGREE LESS THAN  $k$

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**Abstract**

A  $k$ -tree is a tree with maximum degree at most  $k$ . In this paper, we give a degree sum condition for a graph to have a spanning  $k$ -tree in which specified vertices have degree less than  $k$ . We denote by  $\sigma_k(G)$  the minimum value of the degree sum of  $k$  independent vertices in a graph  $G$ . Let  $k \geq 3$  and  $s \geq 0$  be integers, and suppose  $G$  is a connected graph and  $\sigma_k(G) \geq |V(G)| + s - 1$ . Then for any  $s$  specified vertices,  $G$  contains a spanning  $k$ -tree in which every specified vertex has degree less than  $k$ . The degree condition is sharp.

**Keywords:** spanning tree, degree bounded tree, degree sum condition.

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