

## HARARY INDEX OF PRODUCT GRAPHS

K. PATTABIRAMAN AND P. PAULRAJA

*Department of Mathematics  
Annamalai University  
Annamalainagar 608 002, India*

**e-mail:** pramank@gmail.com  
ppraja56@gmail.com

### Abstract

The *Harary index* is defined as the sum of reciprocals of distances between all pairs of vertices of a connected graph. In this paper, the exact formulae for the Harary indices of tensor product  $G \times K_{m_0, m_1, \dots, m_{r-1}}$  and the strong product  $G \boxtimes K_{m_0, m_1, \dots, m_{r-1}}$ , where  $K_{m_0, m_1, \dots, m_{r-1}}$  is the complete multipartite graph with partite sets of sizes  $m_0, m_1, \dots, m_{r-1}$  are obtained. Also upper bounds for the Harary indices of tensor and strong products of graphs are established. Finally, the exact formula for the Harary index of the wreath product  $G \circ G'$  is obtained.

**Keywords:** tensor product, strong product, wreath product, Harary index.

**2010 Mathematics Subject Classification:** 05C12, 05C76.

### REFERENCES

- [1] N. Alon and E. Lubetzky, *Independent set in tensor graph powers*, J. Graph Theory **54** (2007) 73–87.  
doi:10.1002/jgt.20194
- [2] A.M. Assaf, *Modified group divisible designs*, Ars Combin. **29** (1990) 13–20.
- [3] R. Balakrishnan and K. Ranganathan, *A Textbook of Graph Theory*, Second Edition (Springer, New York, 2012).  
doi:10.1007/978-1-4614-4529-6
- [4] B. Brešar, W. Imrich, S. Klavžar and B. Zmazek, *Hypercubes as direct products*, SIAM J. Discrete Math. **18** (2005) 778–786.  
doi:10.1137/S0895480103438358
- [5] K.C. Das, B. Zhou and N. Trinajstić, *Bounds on Harary index*, J. Math. Chem. **46** (2009) 1377–1393.  
doi:10.1007/s10910-009-9522-8

- [6] J. Devillers and A.T. Balaban, (Eds), *Topological Indices and Related Descriptors in QSAR and QSPR* (Gordon and Breach, Amsterdam, 1999).
- [7] M.V. Diudea, *Indices of reciprocal properties or Harary indices*, J. Chem. Inf. Comput. Sci. **37** (1997) 292–299.  
doi:10.1021/ci960037w
- [8] L. Feng and A. Ilić, *Zagreb, Harary and hyper-Wiener indices of graphs with a given matching number*, Appl. Math. Lett. **23** (2010) 943–948.  
doi:10.1016/j.aml.2010.04.017
- [9] I. Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry* (Springer-Verlag, Berlin, 1986).  
doi:10.1007/978-3-642-70982-1
- [10] I. Gutman, *A property of the Wiener number and its modifications*, Indian J. Chem. **36A** (1997) 128–132.
- [11] R. Hammack, W. Imrich and S. Klavžar, *Handbook of Product Graphs* (CRC Press, New York, 2011).
- [12] M. Hoji, Z. Luo and E. Vumar, *Wiener and vertex PI indices of Kronecker products of graphs*, Discrete Appl. Math. **158** (2010) 1848–1855.  
doi:10.1016/j.dam.2010.06.009
- [13] O. Ivanciuc, T.S. Balaban and A.T. Balaban, *Reciprocal distance matrix, related local vertex invariants and topological indices*, J. Math. Chem. **12** (1993) 309–318.  
doi:10.1007/BF01164642
- [14] M.H. Khalifeh, H. Youseri-Azari and A.R. Ashrafi, *Vertex and edge PI indices of Cartesian product of graphs*, Discrete Appl. Math. **156** (2008) 1780–1789.  
doi:10.1016/j.dam.2007.08.041
- [15] B. Lučić, A. Miličević, S. Nikolić and N. Trinajstić, *Harary index-twelve years later*, Croat. Chem. Acta **75** (2002) 847–868.
- [16] I. Lukovits, *Wiener-type graph invariants*, in: M.V. Diudea (Ed.), *QSPR/QSAR Studies by Molecular Descriptors* (Nova Science Publishers, Huntington, New York, 2001).
- [17] A. Mamut and E. Vumar, *Vertex vulnerability parameters of Kronecker products of complete graphs*, Inform. Process. Lett. **106** (2008) 258–262.  
doi:10.1016/j.ipl.2007.12.002
- [18] D.E. Needham, I.C. Wei and P.G. Seybold, *Molecular modeling of the physical properties of alkanes*, J. Amer. Chem. Soc. **110** (1988) 4186–4194.  
doi:10.1021/ja00221a015
- [19] K. Pattabiraman and P. Paulraja, *On some topological indices of the tensor product of graphs*, Discrete Appl. Math. **160** (2012) 267–279.  
doi:10.1016/j.dam.2011.10.020
- [20] K. Pattabiraman and P. Paulraja, *Wiener and vertex PI indices of the strong product of graphs*, Discuss. Math. Graph Theory **32** (2012) 749–769.  
doi:10.7151/dmgt.1647

- [21] K. Pattabiraman and P. Paulraja, *Wiener index of the tensor product of a path and a cycle*, Discuss. Math. Graph Theory **31** (2011) 737–751.  
doi:10.7151/dmgt.1576
- [22] D. Plavsić, S. Nikolić, N. Trinajstić and Z. Mihalić, *On the Harary index for the characterization of chemical graphs*, J. Math. Chem. **12** (1993) 235–250.
- [23] R. Todeschini and V. Consonni, Handbook of Molecular Descriptors (Wiley-VCH, Weinheim, 2000).
- [24] N. Trinajstić, S. Nikolić, S.C. Basak and I. Lukovits, *Distance indices and their hyper-counterparts: Intercorrelation and use in the structure-property modeling, SAR and QSAR in Environmental Research* **12** (2001) 31–54.  
doi:10.1080/10629360108035370
- [25] K. Xu and K.C. Das, *On Harary index of graphs*, Discrete. Appl. Math. **159** (2011) 1631–1640.  
doi:10.1016/j.dam.2011.06.003
- [26] H. Yousefi-Azari, M.H. Khalifeh and A.R. Ashrafi, *Calculating the edge Wiener and edge Szeged indices of graphs*, J. Comput. Appl. Math. **235** (2011) 4866–4870.  
doi:10.1016/j.cam.2011.02.019
- [27] B. Zhou, Z. Du and N. Trinajstić, *Harary index of landscape graphs*, Int. J. Chem. Model. **1** (2008) 35–44.
- [28] B. Zhou, X. Cai and N. Trinajstić, *On the Harary index*, J. Math. Chem. **44** (2008) 611–618.  
doi:10.1007/s10910-007-9339-2

Received 4 March 2013  
Revised 14 October 2013  
Accepted 24 December 2013