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HARARY INDEX OF PRODUCT GRAPHS

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Abstract

The *Harary index* is defined as the sum of reciprocals of distances between all pairs of vertices of a connected graph. In this paper, the exact formulae for the Harary indices of tensor product $G \times K_{m_0,m_1,\ldots,m_{r-1}}$ and the strong product $G \boxtimes K_{m_0,m_1,\ldots,m_{r-1}}$, where $K_{m_0,m_1,\ldots,m_{r-1}}$ is the complete multipartite graph with partite sets of sizes $m_0, m_1, \ldots, m_{r-1}$ are obtained. Also upper bounds for the Harary indices of tensor and strong products of graphs are estabilished. Finally, the exact formula for the Harary index of the wreath product $G \circ G'$ is obtained.

Keywords: tensor product, strong product, wreath product, Harary index.

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