

α -LABELINGS OF A CLASS OF GENERALIZED PETERSEN GRAPHS

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Abstract

An α -labeling of a bipartite graph Γ of size e is an injective function $f : V(\Gamma) \rightarrow \{0, 1, 2, \dots, e\}$ such that $\{|f(x) - f(y)| : [x, y] \in E(\Gamma)\} = \{1, 2, \dots, e\}$ and with the property that its maximum value on one of the two bipartite sets does not reach its minimum on the other one. We prove that the generalized Petersen graph $P_{8n,3}$ admits an α -labeling for any integer $n \geq 1$ confirming that the conjecture posed by Vietri in [10] is true. In such a way we obtain an infinite class of decompositions of complete graphs into copies of $P_{8n,3}$.

Keywords: generalized Petersen graph, α -labeling, graph decomposition.

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