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A NOTE ON VERTEX COLORINGS OF PLANE GRAPHS[1]

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Abstract

Given an integer valued weighting of all elements of a 2-connected plane graph G with vertex set V, let $c(v)$ denote the sum of the weight of $v \in V$ and of the weights of all edges and all faces incident with v. This vertex coloring of G is *proper* provided that $c(u) \neq c(v)$ for any two adjacent vertices u and v of G. We show that for every 2-connected plane graph there is such a proper vertex coloring with weights in $\{1, 2, 3\}$. In a special case, the value 3 is improved to 2.

Keywords: plane graph, vertex coloring.

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