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SOME REMARKS ON THE STRUCTURE OF STRONG k-TRANSITIVE DIGRAPHS[1]

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Abstract

A digraph D is k-transitive if the existence of a directed path $(v_0, v_1, \ldots, v_k)$, of length k implies that $(v_0, v_k) \in A(D)$. Clearly, a 2-transitive digraph is a transitive digraph in the usual sense. Transitive digraphs have been characterized as compositions of complete digraphs on an acyclic transitive digraph. Also, strong 3 and 4-transitive digraphs have been characterized.

In this work we analyze the structure of strong k-transitive digraphs having a cycle of length at least k. We show that in most cases, such digraphs are complete digraphs or cycle extensions. Also, the obtained results are used to prove some particular cases of the Laborde-Payan-Xuong Conjecture.

Keywords: digraph, transitive digraph, k-transitive digraph, quasi-transitive digraph, k-quasi-transitive digraph, Laborde-Payan-Xuong Conjecture.

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