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DOWNHILL DOMINATION IN GRAPHS

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Abstract

A path $\pi = (v_1, v_2, \ldots, v_{k+1})$ in a graph $G = (V, E)$ is a *downhill path* if for every i, $1 \leq i \leq k$, $\deg(v_i) \geq \deg(v_{i+1})$, where $\deg(v_i)$ denotes the degree of vertex $v_i \in V$. The *downhill domination number* equals the minimum cardinality of a set $S \subseteq V$ having the property that every vertex $v \in V$ lies on a downhill path originating from some vertex in S. We investigate downhill domination numbers of graphs and give upper bounds. In particular, we show that the downhill domination number of a graph is at most half its order, and that the downhill domination number of a tree is at most one third its order. We characterize the graphs obtaining each of these bounds.

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