

A CHARACTERIZATION OF 2-TREE PROBE INTERVAL GRAPHS

DAVID E. BROWN

*Department of Mathematics and Statistics
Utah State University
Logan, UT 84322, USA*

e-mail: david.e.brown@usu.edu

BREEANN M. FLESCH

*Mathematics Department
Western Oregon University
Monmouth, OR 97361, USA*

e-mail: fleschb@wou.edu

AND

J. RICHARD LUNDGREN

*Department of Mathematical Sciences
University of Colorado Denver
Denver, CO 80217, USA*

e-mail: richard.lundgren@ucdenver.edu

Abstract

A graph is a *probe interval graph* if its vertices correspond to some set of intervals of the real line and can be partitioned into sets P and N so that vertices are adjacent if and only if their corresponding intervals intersect and at least one belongs to P . We characterize the 2-trees which are probe interval graphs and extend a list of forbidden induced subgraphs for such graphs created by Pržulj and Corneil in [2-tree probe interval graphs have a large obstruction set, Discrete Appl. Math. **150** (2005) 216–231].

Keywords: interval graph, probe interval graph, 2-tree.

2010 Mathematics Subject Classification: 05C62, 05C75.

REFERENCES

- [1] L.W. Beineke, *Characterizations of derived graphs*, J. Combin. Theory **9** (1970) 129–135.
doi:10.1016/S0021-9800(70)80019-9
- [2] L.W. Beineke and R.E. Pippert, *Properties and characterizations of k -trees*, Mathematika **18** (1971) 141–151.
doi:10.1112/S0025579300008500
- [3] D.E. Brown, Variations on Interval Graphs, PhD Thesis, (Univ. of Colorado, Denver, USA, 2004).
- [4] D.E. Brown, A.H. Busch and G. Isaak, *Linear time recognition algorithms and structure theorems for bipartite tolerance graphs and bipartite probe interval graphs*, Discrete Math. Theor. Comput. Sci. **12**(5) (2010) 63–82.
- [5] D.E. Brown, S.C. Flink and J.R. Lundgren, *Interval k -graphs*, in: Proceedings of the Thirty-third Southeastern International Conference on Combinatorics, Graph Theory and Computing (Boca Raton, FL, 2002) **156** (2002) 5–16.
- [6] D.E. Brown and L.J. Langley, *Forbidden subgraph characterizations of bipartite unit probe interval graphs*, Australas. J. Combin. **52** (2012) 19–31.
- [7] D.E. Brown and J.R. Lundgren, *Bipartite probe interval graphs, circular arc graphs, and interval point bigraphs*, Australas. J. Combin. **35** (2006) 221–236.
- [8] D.E. Brown, J.R. Lundgren and L. Sheng, *A characterization of cycle-free unit probe interval graphs*, Discrete Appl. Math. **157** (2009) 762–767.
doi:10.1016/j.dam.2008.07.004
- [9] P.C. Fishburn, Interval Orders and Interval Graphs (Wiley & Sons, 1985).
- [10] B. Flesch and J.R. Lundgren, *A characterization of k -trees that are interval p -graphs*, Australas. J. Combin. **49** (2011) 227–237.
- [11] D.R. Fulkerson and O.A. Gross, *Incidence matrices and interval graphs*, Pacific J. Math. **15** (1965) 835–855.
doi:10.2140/pjm.1965.15.835
- [12] M.C. Golumbic, Algorithmic Graph Theory and Perfect Graphs (Academic Press, New York, 1980).
- [13] M.C. Golumbic and M. Lipshteyn, *On the hierarchy of interval, probe and tolerance graphs*, in: Proceedings of the Thirty-second Southeastern International Conference on Combinatorics, Graph Theory and Computing (Baton Rouge, LA, 2001) **153** (2001) 97–106.
- [14] M.C. Golumbic and A.N. Trenk, Tolerance Graphs, Cambridge Studies in Advanced Mathematics, Vol. **18** (Cambridge University Press, Cambridge, 2004).
- [15] C.G. Lekkerkerker and J.Ch. Boland, *Representation of a finite graph by a set of intervals on the real line*, Fund. Math. **51** (1962/1963) 45–64.

- [16] R. McConnell and Y. Nussbaum, *Linear-time recognition of probe interval graphs*, in: Proceedings of the 17th Annual European Symposium, Algorithms-ESA 2009, Lecture Notes in Comput. Sci. **5757** (2009) 349–360.
doi:10.1007/978-3-642-04128-0_32
- [17] F.R. McMorris, C. Wang and P. Zhang, *On probe interval graphs*, Discrete Appl. Math. **88** (1998) 315–324.
doi:10.1016/S0166-218X(98)00077-8
- [18] A. Proskurowski, *Separating subgraphs in k -trees: cables and caterpillars*, Discrete Math. **49** (1984) 275–285.
doi:10.1016/0012-365X(84)90164-X
- [19] N. Pržulj and D.G. Corneil, *2-tree probe interval graphs have a large obstruction set*, Discrete Appl. Math. **150** (2005) 216–231.
doi:10.1016/j.dam.2004.06.015
- [20] F.S. Roberts, Discrete Mathematical Models (Prentice-Hall, Upper Saddle River, NJ, 1976).
- [21] L. Sheng, *Cycle free probe interval graphs*, in: Proceedings of the Thirtieth South-eastern International Conference on Combinatorics, Graph Theory, and Computing (Boca Raton, FL, 1999) **140** (1999) 33–42.
- [22] P. Zhang, Methods of mapping DNA fragments, United States Patent, 1997
<http://www.cc.columbia.edu/cu/cie/techlists/patents/5667970.htm>
- [23] P. Zhang, E.A. Schon, E. Cayanis, J. Weiss, S. Kistler and P.E. Bourne, *An algorithm based on graph theory for the assembly of contigs in physical mapping of DNA*, Comput. Appl. Biosci. **10(3)** (1994) 309–317.

Received 3 December 2012

Revised 20 May 2013

Accepted 20 May 2013