Discussiones Mathematicae
Graph Theory 34 (2014) 577–584
doi:10.7151/dmgt.1753

ON THE INDEPENDENCE NUMBER OF EDGE CHROMATIC CRITICAL GRAPHS*

Shiyou Pang[1], Lianying Miao[2†]

Wenyao Song[1] and Zhengke Miao[2]

[1]*School of Science, China University of Mining and Technology Xuzhou, Jiangsu, 221008, P.R.China*

[2]*Department of Mathematics, Jiangsu Normal University, Xuzhou, Jiangsu, 221116, P.R.China*

e-mail: 615595479@qq.com
miaolianying@cumt.edu.cn
2408955057@qq.com
zkmiao@163.com

Abstract

In 1968, Vizing conjectured that for any edge chromatic critical graph $G = (V, E)$ with maximum degree Δ and independence number $\alpha(G)$, $\alpha(G) \leq \frac{|V|}{2}$. It is known that $\alpha(G) < \frac{3\Delta-2}{5\Delta-2}|V|$. In this paper we improve this bound when $\Delta \geq 4$. Our precise result depends on the number n_2 of 2-vertices in G, but in particular we prove that $\alpha(G) \leq \frac{3\Delta-3}{5\Delta-3}|V|$ when $\Delta \geq 5$ and $n_2 \leq 2(\Delta - 1)$.

Keywords: edge coloring, edge-chromatic critical graphs, independence number.

2010 Mathematics Subject Classification: 05C15.

*This work was supported by NSFC (11271365, 11171288) and the Fundamental Research Funds for the Central Universitie (2012LWB46).

†Corresponding author.

References

[1] G. Brinkmann, S.A. Choudum, S. Grünewald and E. Steffen, *Bounds for the independence number of critical graphs*, Bull. London Math. Soc. **32** (2000) 137–140. doi:10.1112/S0024609399006645

[2] S. Grünewald and E. Steffen, *Independent sets and* 2*-factors in edge chromatic critical graphs*, J. Graph Theory **45** (2004) 113–118. doi:10.1002/jgt.10141

[3] R. Luo and Y. Zhao, *A note on Vizing's independence number conjecture of edge chromatic critical graphs*, Discrete Math. **306** (2006) 1788–1790. doi:10.1016/j.disc.2006.03.040

[4] R. Luo and Y. Zhao, *An application of Vizing and Vizing-like adjacency lemmas to Vizing's independence number conjecture of edge chromatic critical graphs*, Discrete Math. **309** (2009) 2925–2929. doi:10.1016/j.disc.2008.06.019

[5] R. Luo, L.Y. Miao and Y. Zhao, *The size of edge chromatic critical graphs with maximum degree* 6, J. Graph Theory **60** (2009) 149–171. doi:10.1002/jgt.20351

[6] L.Y. Miao, *On the independence number of edge chromatic critical graphs*, Ars Combin. **98** (2011) 471–481.

[7] D.P. Sanders and Y. Zhao, *Planar graphs with maximum degree seven are Class I*, J. Combin. Theory (B) **83** (2001) 201–212. doi:0.1006/jctb.2001.2047

[8] V.G. Vizing, *On an estimate of the chromatic class of a p-graph*, Diskret. Analiz. **3** (1964) 25–30 (in Russian).

[9] V.G. Vizing, *Some unsolved problems in graph theory*, Uspekhi Mat. Nauk **23** (1968) 117–134, Russian Math. Surveys **23** (1968) 125–142. doi:10.1070/RM1968v023n06ABEH001252

[10] D.R. Woodall, *The independence number of an edge chromatic critical graph*, J. Graph Theory **66** (2011) 98–103. doi:10.1002/jgt.20493

[11] D.R. Woodall, *The average degree of an edge-chromatic critical graph II*, J. Graph Theory **56** (2007) 194–218. doi:10.1002/jgt.20259

[12] H.P. Yap, Some Topics in Graph Theory, London Math. Soc. Lecture Notes Ser. Vol. 108 (Cambridge Univ. Press, 1986). doi:10.1017/CBO9780511662065

[13] L. Zhang, *Every planar graph with maximum degree* 7 *is of class* 1, Graphs Combin. **16** (2000) 467–495. doi:10.1007/s003730070009

Received 22 December 2012
Revised 11 July 2013
Accepted 11 July 2013