

5-STARS OF LOW WEIGHT IN NORMAL PLANE MAPS WITH MINIMUM DEGREE 5

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Abstract

It is known that there are normal plane maps M_5 with minimum degree 5 such that the minimum degree-sum $w(S_5)$ of 5-stars at 5-vertices is arbitrarily large. In 1940, Lebesgue showed that if an M_5 has no 4-stars of cyclic type $(5, 6, 6, 5)$ centered at 5-vertices, then $w(S_5) \leq 68$. We improve this bound of 68 to 55 and give a construction of a $(5, 6, 6, 5)$ -free M_5 with $w(S_5) = 48$.

Keywords: graph, plane map, vertex degree, weight, light subgraph.

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