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ON EULERIAN IRREGULARITY IN GRAPHS

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Abstract

A closed walk in a connected graph G that contains every edge of G exactly once is an Eulerian circuit. A graph is Eulerian if it contains an Eulerian circuit. It is well known that a connected graph G is Eulerian if and only if every vertex of G is even. An Eulerian walk in a connected graph G is a closed walk that contains every edge of G at least once, while an irregular Eulerian walk in G is an Eulerian walk that encounters no two edges of G the same number of times. The minimum length of an irregular Eulerian walk in G is called the Eulerian irregularity of G and is denoted by $EI(G)$. It is known that if G is a nontrivial connected graph of size m, then $\binom{m+1}{2} \leq EI(G) \leq 2\binom{m+1}{2}$. A necessary and sufficient condition has been established for all pairs k, m of positive integers for which there is a nontrivial connected graph G of size m with $EI(G) = k$. A subgraph F in a graph G is an even subgraph of G if every vertex of F is even. We present a formula for the Eulerian irregularity of a graph in terms of the size of certain even subgraph of the graph. Furthermore, Eulerian irregularities are determined for all graphs of cycle rank 2 and all complete bipartite graphs.

Keywords: Eulerian walks, Eulerian irregularity.

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