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FAMILIES OF TRIPLES WITH HIGH MINIMUM DEGREE ARE HAMILTONIAN

Vojtech Rödl[1]

Emory University
Atlanta, GA, USA

e-mail: rodl@mathcs.emory.edu

AND

Andrzej Ruciński[2]

A. Mickiewicz University
Poznań, Poland

e-mail: rucinski@amu.edu.pl

Abstract

In this paper we show that every family of triples, that is, a 3-uniform hypergraph, with minimum degree at least $(\frac{5-\sqrt{5}}{3}+\gamma)\binom{n-1}{2}$ contains a tight Hamiltonian cycle.

Keywords: 3-uniform hypergraph, Hamilton cycle, minimum vertex degree.

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