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THE NICHE GRAPHS OF INTERVAL ORDERS

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Abstract

The *niche graph* of a digraph D is the (simple undirected) graph which has the same vertex set as D and has an edge between two distinct vertices x and y if and only if $N_D^+(x) \cap N_D^+(y) \neq \emptyset$ or $N_D^-(x) \cap N_D^-(y) \neq \emptyset$, where $N_D^+(x)$ (resp. $N_D^-(x)$) is the set of out-neighbors (resp. in-neighbors) of x in D. A digraph $D = (V, A)$ is called a *semiorder* (or a *unit interval order*) if there exist a real-valued function $f : V \to \mathbb{R}$ on the set V and a positive real number $\delta \in \mathbb{R}$ such that $(x, y) \in A$ if and only if $f(x) > f(y) + \delta$. A digraph $D = (V, A)$ is called an *interval order* if there exists an assignment J of a closed real interval $J(x) \subset \mathbb{R}$ to each vertex $x \in V$ such that $(x, y) \in A$ if and only if $\min J(x) > \max J(y)$.

Kim and Roberts characterized the competition graphs of semiorders and interval orders in 2002, and Sano characterized the competition-common enemy graphs of semiorders and interval orders in 2010. In this note, we give characterizations of the niche graphs of semiorders and interval orders.

Keywords: competition graph, niche graph, semiorder, interval order.

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References

[1] C. Cable, K.F. Jones, J.R. Lundgren and S. Seager, *Niche graphs*, Discrete Appl. Math. **23** (1989) 231–241.
doi:10.1016/0166-218X(89)90015-2

[2] J.E. Cohen, *Interval graphs and food webs. A finding and a problem*, RAND Corporation, Document 17696-PR, Santa Monica, California (1968).

[3] P.C. Fishburn, Interval Orders and Interval Graphs: A Study of Partially Ordered Sets, Wiley-Interscience Series in Discrete Mathematics, A Wiley-Interscience Publication (John Wiley & Sons Ltd., Chichester, 1985).

[4] S.-R. Kim and F.S. Roberts, *Competition graphs of semiorders and Conditions $C(p)$ and $C^*(p)$*, Ars Combin. **63** (2002) 161–173.

[5] Y. Sano, *The competition-common enemy graphs of digraphs satisfying conditions $C(p)$ and $C'(p)$*, Congr. Numer. **202** (2010) 187–194.

[6] D.D. Scott, *The competition-common enemy graph of a digraph*, Discrete Appl. Math. **17** (1987) 269–280.
doi:10.1016/0166-218X(87)90030-8

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