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Note

A NOTE ON FACE COLORING ENTIRE WEIGHTINGS OF PLANE GRAPHS[1]

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Abstract

Given a weighting of all elements of a 2-connected plane graph $G = (V, E, F)$, let $f(\alpha)$ denote the sum of the weights of the edges and vertices incident with the face α and also the weight of α. Such an entire weighting is a proper face colouring provided that $f(\alpha) \neq f(\beta)$ for every two faces α and β sharing an edge. We show that for every 2-connected plane graph there is a proper face-colouring entire weighting with weights 1 through 4. For some families we improved 4 to 3.

Keywords: entire weighting, plane graph, face colouring.

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References

[1] L. Addario-Berry, K. Dalal, C. McDiarmid, B.A. Reed and A. Thomason, *Vertex-colouring edge-weightings*, Combinatorica **27** (2007) 1–12. doi:10.1007/s00493-007-0041-6

[2] L. Addario-Berry, K. Dalal and B.A. Reed, *Degree constrainted subgraphs*, Discrete Appl. Math. **156** (2008) 1168–1174. doi:10.1016/j.dam.2007.05.059

[3] K. Appel and W. Haken, *Every planar map is four-colorable,* I. *Discharging*, Illinois J. Math. **21** (1977) 429–490.

[4] M. Bača, S. Jendrol', K.M. Kathiresan and K. Muthugurupackiam, *Entire labeling of plane graph*, (submitted).

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[5] M. Bača, S. Jendrol', M. Miller and J. Ryan, *On irregular total labellings*, Discrete Math. **307** (2007) 1378–1388.
doi:10.1016/j.disc.2005.11.075

[6] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer-Verlag, Heidelberg, 2008).

[7] A.J. Dong and G.H. Wang, *Neighbor sum distinguishing colorings of some graphs*, Discrete Math. Algorithms Appl. (2012) **4(4)** 1250047.
doi:10.1142/S1793830912500474

[8] E. Flandrin, J.F. Saclé, A. Marczyk, J. Przybyło and M. Woźniak, *Neighbor sum distinguishing index*, Graphs Combin. **29** (2013) 1329–1336.
doi:10.1007/s00373-012-1191-x

[9] A. Frieze, R.J. Gould, M. Karoński and F. Pfender, *On graph irregularity strenght*, J. Graph Theory **41** (2002) 120–137.
doi:10.1002/jgt.10056

[10] H. Grötzsch, *Zur Theorie der discreten Gebilde. VII. Ein Dreifarbensatz für dreikreisfreie Netze auf der Kugel.*, Wiss. Z. Martin-Luther-Univ. Halle-Wittenberg, Math.-Mat. Reihe **8** (1958/1959) 109–120.

[11] G. Chartrand, M.S. Jacobson, L. Lehel, O.R. Oellermann, S. Ruiz and F. Saba, *Irregular networks*, Congr. Numer. **64** (1988) 187–192.

[12] M. Kalkowski, *A note on* $1,2$*-conjecture*, Electron. J. Combin. (to appear).

[13] M. Kalkowski, M. Karoński and F. Pfender, *Vertex-coloring edge-weightings: towards the* 1-2-3*-conjecture*, J. Combin. Theory (B) **100** (2010) 347–349.
doi:10.1016/j.jctb.2009.06.002

[14] M. Karoński, T. Łuczak and A. Thomason, *Edge weights and vertex colours*, J. Combin. Theory (B) **91** (2004) 151–157.
doi:10.1016/j.jctb.2003.12.001

[15] J. Przybyło and M. Woźniak, *On* $1,2$ *conjecture*, Discrete Math. Theor. Comput. Sci. **12** (2010) 101–108.

[16] T. Wang and Q. Yu, *On vertex-coloring* 13*-edge-weighting*, Front. Math. China **3** (2008) 1–7.
doi:10.1007/s11464-008-0041-x

[17] W. Wang and X. Zhu, *Entire colouring of plane graphs*, J. Combin. Theory (B) **101** (2011) 490–501.
doi:10.1016/j.jctb.2011.02.006

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