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HEAVY SUBGRAPH PAIRS FOR TRACEABILITY OF BLOCK-CHAINS

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Abstract

A graph is called traceable if it contains a Hamilton path, i.e., a path containing all its vertices. Let G be a graph on n vertices. We say that an induced subgraph of G is o_{-1}-heavy if it contains two nonadjacent vertices which satisfy an Ore-type degree condition for traceability, i.e., with degree sum at least $n-1$ in G. A block-chain is a graph whose block graph is a path, i.e., it is either a P_1, P_2, or a 2-connected graph, or a graph with at least one cut vertex and exactly two end-blocks. Obviously, every traceable graph is a block-chain, but the reverse does not hold. In this paper we characterize all the pairs of connected o_{-1}-heavy graphs that guarantee traceability of block-chains. Our main result is a common extension of earlier work on degree sum conditions, forbidden subgraph conditions and heavy subgraph conditions for traceability.

Keywords: o_{-1}-heavy subgraph, block-chain traceable graph, Ore-type condition, forbidden subgraph.

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References

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[1] P. Bedrossian, G. Chen, and R.H. Schelp, *A generalization of Fan's condition for hamiltonicity, pancyclicity, and hamiltonian connectedness*, Discrete Math. **115** (1993) 39–50.
doi:10.1016/0012-365X(93)90476-A

[2] J.A. Bondy and U.S.R. Murty, Graph Theory, Springer Graduate Texts in Mathematics **244** (2008).

[3] H.J. Broersma, Z. Ryjáček and I. Schiermeyer, *Dirac's minimum degree condition restricted to claws*, Discrete Math. **167/168** (1997) 155–166.
doi:10.1016/S0012-365X(96)00224-5

[4] H.J. Broersma and H.J. Veldman, *Restrictions on induced subgraphs ensuring hamiltonicity or pancyclicity of $K_{1,3}$-free graphs*, in: R. Bodendiek (Ed.) Contemporary Methods in Graph Theory, (BI Wissenschaftsverlag, Mannheim-Wien-Zfirich, 1990) 181–194.

[5] R. Čada, *Degree conditions on induced claws*, Discrete Math. **308** (2008) 5622–5631.
doi:10.1016/j.disc.2007.10.026

[6] D. Duffus, R.J. Gould and M.S. Jacobson, *Forbidden subgraphs and the hamiltonian theme*, in: The Theory and Applications of Graphs (Kalamazoo, Mich. 1980, Wiley, New York, 1981) 297–316.

[7] R.J. Faudree and R.J. Gould, *Characterizing forbidden pairs for Hamiltonian properties*, Discrete Math. **173** (1997) 45–60.
doi:10.1016/S0012-365X(96)00147-1

[8] H. Fleischner, *The square of every two-connected graph is hamiltonian*, J. Combin. Theory (B) **16** (1974) 29–34.
doi:10.1016/0095-8956(74)90091-4

[9] B. Li, H.J. Broersma and S. Zhang, *Forbidden subgraph pairs for traceability of block-chains*, Electron. J. Graph Theory Appl. **1** (2013) 1–10.

[10] B. Li, Z. Ryjáček, Y. Wang and S. Zhang, *Pairs of heavy subgraphs for Hamiltonicity of 2-connected graphs*, SIAM J. Discrete Math. **26** (2012) 1088–1103.
doi:10.1137/11084786X

[11] B. Li and S. Zhang, *On traceability of claw-o_{-1}-heavy graphs* (2013).
arXiv:1303.0991v1

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