Discussiones Mathematicae
Graph Theory 34 (2014) 233–247
doi:10.7151/dmgt.1736

THE DEPRESSION OF A GRAPH AND k-KERNELS

Mark Schurch and Christine Mynhardt

Department of Mathematics and Statistics
University of Victoria, P.O. Box 3060 STN CSC
Victoria, BC, Canada V8W 3R4

e-mail: mschurch@uvic.ca
mynhardt@math.uvic.ca

Abstract

An *edge ordering* of a graph G is an injection $f : E(G) \to \mathbb{R}$, the set of real numbers. A path in G for which the edge ordering f increases along its edge sequence is called an *f-ascent*; an f-ascent is *maximal* if it is not contained in a longer f-ascent. The *depression* of G is the smallest integer k such that any edge ordering f has a maximal f-ascent of length at most k. A k-kernel of a graph G is a set of vertices $U \subseteq V(G)$ such that for any edge ordering f of G there exists a maximal f-ascent of length at most k which neither starts nor ends in U. Identifying a k-kernel of a graph G enables one to construct an infinite family of graphs from G which have depression at most k. We discuss various results related to the concept of k-kernels, including an improved upper bound for the depression of trees.

Keywords: edge ordering of a graph, increasing path, monotone path, depression.

2010 Mathematics Subject Classification: 05C78, 05C38.

References

[1] A. Bialostocki and Y. Roditty, *A monotone path in an edge-ordered graph*, Int. J. Math. Math. Sci. **10** (1987) 315–320.
doi:10.1155/S0161171287000383

[2] A.P. Burger, E.J. Cockayne and C.M. Mynhardt, *Altitude of small complete and complete bipartite graphs*, Australas. J. Combin. **31** (2005) 167–177.

[3] A.R. Calderbank and F.R.K. Chung and D.G. Sturtevant, *Increasing sequences with nonzero block sums and increasing paths in edge-ordered graphs*, Discrete Math. **50** (1984) 15–28.
doi:10.1016/0012-365X(84)90031-1

[4] G. Chartrand and L.M. Lesniak, Graphs and Digraphs (Third Ed.) (Wadsworth, 1996).

[5] V. Chvátal and J. Komlós, *Some combinatorial theorems on monotonicity*, Canad. Math. Bull. **14** (1971) 151–157.
doi:10.4153/CMB-1971-028-8

[6] T.C. Clark and B. Falvai, N.D.R. Henderson and C.M. Mynhardt, *Altitude of* 4*-regular circulants*, AKCE Int. J. Graphs Comb. **1** (2004) 149–166.

[7] E.J. Cockayne and G. Geldenhuys, P.J.P Grobler, C.M. Mynhardt and J.H. van Vuuren, *The depression of a graph*, Util. Math. **69** (2006) 143–160.

[8] E.J. Cockayne and C.M. Mynhardt, *Altitude of* $K_{3,n}$, J. Combin. Math. Combin. Comput. **52** (2005) 143–157.

[9] E.J. Cockayne and C.M. Mynhardt, *A lower bound for the depression of trees*, Australas. J. Combin. **35** (2006) 319–328.

[10] T. Dzido and H. Furmańczyk, *Altitude of wheels and wheel-like graphs*, Cent. Eur. J. Math. **8** (2010) 319–326.
doi:10.2478/s11533-010-0017-4

[11] I. Gaber-Rosenblum and Y. Roditty, *The depression of a graph and the diameter of its line graph*, Discrete Math. **309** (2009) 1774–1778.
doi:10.1016/j.disc.2008.03.002

[12] R.L. Graham and D.J. Kleitman, *Increasing paths in edge ordered graphs*, Period. Math. Hungar. **3** (1973) 141–148.
doi:10.1007/BF02018469

[13] J. Katrenič and G. Semanišin, *Finding monotone paths in edge-ordered graphs*, Discrete Appl. Math. **158** (2010) 1624–1632.
doi:10.1016/j.dam.2010.05.018

[14] C.M. Mynhardt, *Trees with depression three*, Discrete Math. **308** (2008) 855–864.
doi:10.1016/j.disc.2007.07.039

[15] C.M. Mynhardt, A.P. Burger and T.C. Clark, B. Falvai, and N.D.R. Henderson, *Altitude of regular graphs with girth at least five*, Discrete Math. **294** (2005) 241–257.
doi:10.1016/j.disc.2005.02.007

[16] C.M. Mynhardt and M. Schurch, *A class of graphs with depression three*, Discrete Math. **313** (2013) 1224–1232.
doi:10.1016/j.disc.2012.05.011

[17] Y. Roditty, B. Shoham and R. Yuster, *Monotone paths in edge-ordered sparse graphs*, Discrete Math. **226** (2001) 411–417.
doi:10.1016/S0012-365X(00)00174-6

[18] R. Yuster, *Large monotone paths in graphs with bounded degree*, Graphs Combin. **17** (2001) 579–587.
doi:10.1007/s003730170031

Received 5 November 2012
Revised 26 February 2013
Accepted 26 February 2013