

THE DEPRESSION OF A GRAPH AND k -KERNELS

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Abstract

An *edge ordering* of a graph G is an injection $f : E(G) \rightarrow \mathbb{R}$, the set of real numbers. A path in G for which the edge ordering f increases along its edge sequence is called an *f -ascent*; an *f -ascent* is *maximal* if it is not contained in a longer *f -ascent*. The *depression* of G is the smallest integer k such that any edge ordering f has a maximal *f -ascent* of length at most k . A *k -kernel* of a graph G is a set of vertices $U \subseteq V(G)$ such that for any edge ordering f of G there exists a maximal *f -ascent* of length at most k which neither starts nor ends in U . Identifying a *k -kernel* of a graph G enables one to construct an infinite family of graphs from G which have depression at most k . We discuss various results related to the concept of *k -kernels*, including an improved upper bound for the depression of trees.

Keywords: edge ordering of a graph, increasing path, monotone path, depression.

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