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THE IRREGULARITY OF GRAPHS UNDER GRAPH OPERATIONS

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Abstract

The *irregularity* of a simple undirected graph G was defined by Albertson [5] as $\mathrm{irr}(G) = \sum_{uv \in E(G)} |d_G(u) - d_G(v)|$, where $d_G(u)$ denotes the degree of a vertex $u \in V(G)$. In this paper we consider the irregularity of graphs under several graph operations including join, Cartesian product, direct product, strong product, corona product, lexicographic product, disjunction and symmetric difference. We give exact expressions or (sharp) upper bounds on the irregularity of graphs under the above mentioned operations.

Keywords: irregularity of graphs, total irregularity of graphs, graph operations, Zagreb indices.

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