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ON THE UNIQUENESS OF D-VERTEX MAGIC CONSTANT

S. Arumugam[1], N. Kamatchi[2]

National Centre for Advanced Research
in Discrete Mathematics (n-CARDMATH)
Kalasalingam University
Anand Nagar, Krishnankoil-626 126, Tamil Nadu, India

e-mail: s.arumugam.klu@gmail.com
n_kamatchi@yahoo.com

AND

G.R. Vijayakumar

School of Mathematics
Tata Institute of Fundamental Research
Homi Bhabha Road, Colaba, Mumbai 400 005, India

e-mail: vijay@math.tifr.res.in

Abstract

Let $G = (V, E)$ be a graph of order n and let $D \subseteq \{0, 1, 2, 3, \ldots\}$. For $v \in V$, let $N_D(v) = \{u \in V : d(u, v) \in D\}$. The graph G is said to be D-vertex magic if there exists a bijection $f : V(G) \to \{1, 2, \ldots, n\}$ such that for all $v \in V$, $\sum_{u \in N_D(v)} f(u)$ is a constant, called D-vertex magic constant. O'Neal and Slater have proved the uniqueness of the D-vertex magic constant by showing that it can be determined by the D-neighborhood fractional domination number of the graph. In this paper we give a simple and elegant proof of this result. Using this result, we investigate the existence of distance magic labelings of complete r-partite graphs where $r \geq 4$.

Keywords: distance magic graph, D-vertex magic graph, magic constant, dominating function, fractional domination number.

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[1] Also at School of Electrical Engineering and Computer Science, The University of Newcastle, NSW 2308, Australia; Department of Computer Science, Liverpool Hope University, Liverpool, UK; Department of Computer Science, Ball State University, USA.

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