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ON THE COMPLEXITY OF THE 3-KERNEL PROBLEM IN SOME CLASSES OF DIGRAPHS[1]

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Abstract

Let D be a digraph with the vertex set $V(D)$ and the arc set $A(D)$. A subset N of $V(D)$ is *k-independent* if for every pair of vertices $u, v \in N$, we have $d(u,v), d(v,u) \geq k$; it is *l-absorbent* if for every $u \in V(D) - N$ there exists $v \in N$ such that $d(u,v) \leq l$. A *k-kernel* of D is a k-independent and $(k-1)$-absorbent subset of $V(D)$. A 2-kernel is called a *kernel*.

It is known that the problem of determining whether a digraph has a kernel ("the kernel problem") is NP-complete, even in quite restricted families of digraphs. In this paper we analyze the computational complexity of the corresponding 3-kernel problem, restricted to three natural families of digraphs.

As a consequence of one of our main results we prove that the kernel problem remains NP-complete when restricted to 3-colorable digraphs.

Keywords: kernel, 3-kernel, NP-completeness, multipartite tournament, cyclically 3-partite digraphs, k-quasi-transitive digraph.

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References

[1] J. Bang-Jensen and G. Gutin, Digraphs (Springer-Verlag, Berlin Heidelberg New York, 2002).

[2] J. Bang-Jensen and J. Huang, *Quasi-transitive digraphs*, J. Graph Theory **20** (1995) 141–161.
doi:10.1002/jgt.3190200205

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[3] C. Berge, Graphs (North-Holland, Amsterdam, 1985).

[4] C. Berge and P. Duchet, *Recent problems and results about kernels in directed graphs*, Discrete Math. **86** (1990) 27–31.
doi:10.1016/0012-365X(90)90346-J

[5] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer-Verlag, Berlin Heidelberg New York, 2008).

[6] V. Chvátal, *On the computational complexity of finding a kernel*, Technical Report Centre de Recherches Mathématiques, Université de Montréal CRM-300 (1973).

[7] A.S. Fraenkel, *Planar kernel and Grundy with* $d \leq 3, d^+ \leq 2, d^- \leq 2$ *are NP-complete*, Discrete Appl. Math. **3** (1981) 257–262.
doi:10.1016/0166-218X(81)90003-2

[8] H. Galeana-Sánchez, I.A. Goldfeder and I. Urrutia, *On the structure of* 3*-quasi-transitive digraphs*, Discrete Math. **310** (2010) 2495–2498.
doi:10.1016/j.disc.2010.06.008

[9] H. Galeana-Sánchez and C. Hernández-Cruz, *k-kernels in multipartite tournaments*, AKCE Int. J. Graphs Comb. **8** (2011) 181–198.

[10] H. Galeana-Sánchez and C. Hernández-Cruz, *On the existence of k-kernels digraphs and in weighted digraphs*, AKCE Int. J. Graphs Comb. **7** (2010) 201–215.

[11] H. Galeana-Sánchez and C. Hernández-Cruz, *Cyclically k-partite digraphs and k-kernels*, Discuss. Math. Graph Theory **31** (2011) 63–78.
doi:10.7151/dmgt.1530

[12] H. Galeana-Sánchez and C. Hernández-Cruz, *k-kernels in generalizations of transitive digraphs*, Discuss. Math. Graph Theory **31** (2011) 293–312.
doi:10.7151/dmgt.1546

[13] H. Galeana-Sánchez, C. Hernández-Cruz and M.A. Juárez-Camacho, *On the existence and number of* $(k+1)$*-kings in* k*-quasi-transitive digraphs*, Discrete Math. **313** (2013) 2582–2591.

[14] C. Hernández-Cruz and H. Galeana-Sánchez, *k-kernels in k-transitive and k-quasi-transitive digraphs*, Discrete Math. **312** (2012) 2522-2530.
doi:10.1016/j.disc.2012.05.005

[15] M. Kwaśnik, A. Włoch and I. Włoch, *Some remarks about* (k, l)*-kernels in directed and undirected graphs*, Discuss. Math. **13** (1993) 29–37.

[16] M. Richardson, *On weakly ordered systems*, Bull. Amer. Math. Soc. **52(2)** (1946) 113–116.
doi:10.1090/S0002-9904-1946-08518-3

[17] A. Sánchez-Flores, *A counterexample to a generalization of Richardson's theorem*, Discrete Math. **65** (1987) 319–320.
doi:10.1016/0012-365X(87)90064-1

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