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MOTION PLANNING IN CARTESIAN PRODUCT GRAPHS

Biswajit Deb[1]

Department of Mathematics
Sikkim Manipal Institute of Technology, India

e-mail: bdeb@iitg.ernet.in

and

Kalpesh Kapoor

Department of Mathematics
Indian Institute of Technology Guwahati, India

e-mail: kalpesh@iitg.ernet.in

Abstract

Let G be an undirected graph with n vertices. Assume that a robot is placed on a vertex and $n-2$ obstacles are placed on the other vertices. A vertex on which neither a robot nor an obstacle is placed is said to have a hole. Consider a single player game in which a robot or obstacle can be moved to adjacent vertex if it has a hole. The objective is to take the robot to a fixed destination vertex using minimum number of moves. In general, it is not necessary that the robot will take a shortest path between the source and destination vertices in graph G. In this article we show that the path traced by the robot coincides with a shortest path in case of Cartesian product graphs. We give the minimum number of moves required for the motion planning problem in Cartesian product of two graphs having girth 6 or more. A result that we prove in the context of Cartesian product of P_n with itself has been used earlier to develop an approximation algorithm for (n^2-1)-puzzle.

Keywords: robot motion in a graph, Cartesian product of graphs.

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