Discussiones Mathematicae
Graph Theory 34 (2014) 207–221
doi:10.7151/dmgt.1726

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DMGT Page

MOTION PLANNING IN CARTESIAN PRODUCT GRAPHS

Biswajit Deb[1]

Department of Mathematics
Sikkim Manipal Institute of Technology, India

e-mail: bdeb@iitg.ernet.in

and

Kalpesh Kapoor

Department of Mathematics
Indian Institute of Technology Guwahati, India

e-mail: kalpesh@iitg.ernet.in

Abstract

Let G be an undirected graph with n vertices. Assume that a robot is placed on a vertex and $n-2$ obstacles are placed on the other vertices. A vertex on which neither a robot nor an obstacle is placed is said to have a hole. Consider a single player game in which a robot or obstacle can be moved to adjacent vertex if it has a hole. The objective is to take the robot to a fixed destination vertex using minimum number of moves. In general, it is not necessary that the robot will take a shortest path between the source and destination vertices in graph G. In this article we show that the path traced by the robot coincides with a shortest path in case of Cartesian product graphs. We give the minimum number of moves required for the motion planning problem in Cartesian product of two graphs having girth 6 or more. A result that we prove in the context of Cartesian product of P_n with itself has been used earlier to develop an approximation algorithm for (n^2-1)-puzzle.

Keywords: robot motion in a graph, Cartesian product of graphs.

2010 Mathematics Subject Classification: 05C85, 05C75, 68R10, 91A43.

[1]This research was done while the author was a Ph.D. student at the Department of Mathematics, Indian Institute of Technology Guwahati, India. The author thanks Sikkim Manipal Institute of Technology and the All India Council for Technical Education (AICTE) to give paid-leave and provide financial support, respectively, for this study.

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Received 10 April 2012
Revised 5 February 2013
Accepted 5 February 2013