Discussiones Mathematicae
Graph Theory 34 (2014) 5–22
doi:10.7151/dmgt.1725

ON MONOCHROMATIC SUBGRAPHS OF EDGE-COLORED COMPLETE GRAPHS

Eric Andrews[1], Futaba Fujie[2]

Kyle Kolasinski[1], Chira Lumduanhom[1]

and

Adam Yusko[1]

[1] *Department of Mathematics*
Western Michigan University
Kalamazoo, MI 49008 USA

[2] *Graduate School of Mathematics*
Nagoya University
Nagoya, Japan 464-8602

e-mail: eric.s.andrews@wmich.edu
futaba@math.nagoya-u.ac.jp
kyle.c.kolasinski@wmich.edu
chira.lumduanhom@wmich.edu
adam.m.yusko@wmich.edu

Abstract

In a red-blue coloring of a nonempty graph, every edge is colored red or blue. If the resulting edge-colored graph contains a nonempty subgraph G without isolated vertices every edge of which is colored the same, then G is said to be monochromatic.

For two nonempty graphs G and H without isolated vertices, the monochromatic Ramsey number $\mathrm{mr}(G, H)$ of G and H is the minimum integer n such that every red-blue coloring of K_n results in a monochromatic G or a monochromatic H. Thus, the standard Ramsey number of G and H is bounded below by $\mathrm{mr}(G, H)$. The monochromatic Ramsey numbers of graphs belonging to some common classes of graphs are studied.

We also investigate another concept closely related to the standard Ramsey numbers and monochromatic Ramsey numbers of graphs. For a fixed integer $n \geq 3$, consider a nonempty subgraph G of order at most n containing no isolated vertices. Then G is a common monochromatic subgraph of K_n if every red-blue coloring of K_n results in a monochromatic copy of

G. Furthermore, G is a maximal common monochromatic subgraph of K_n if G is a common monochromatic subgraph of K_n that is not a proper subgraph of any common monochromatic subgraph of K_n. Let $\mathcal{S}(n)$ and $\mathcal{S}^*(n)$ be the sets of common monochromatic subgraphs and maximal common monochromatic subgraphs of K_n, respectively. Thus, $G \in \mathcal{S}(n)$ if and only if $R(G,G) = \mathrm{mr}(G,G) \leq n$. We determine the sets $\mathcal{S}(n)$ and $\mathcal{S}^*(n)$ for $3 \leq n \leq 8$.

Keywords: Ramsey number, monochromatic Ramsey number, common monochromatic subgraph, maximal common monochromatic subgraph.

2010 Mathematics Subject Classification: 05C15, 05C35, 05C55.

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Received 12 March 2012
Revised 26 September 2012
Accepted 2 October 2012