

TREE-LIKE PARTIAL HAMMING GRAPHS

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Abstract

Tree-like partial cubes were introduced in [B. Brešar, W. Imrich, S. Klavžar, *Tree-like isometric subgraphs of hypercubes*, Discuss. Math. Graph Theory, 23 (2003), 227–240] as a generalization of median graphs. We present some incorrectnesses from that article. In particular we point to a gap in the proof of the theorem about the dismantlability of the cube graph of a tree-like partial cube and give a new proof of that result, which holds also for a bigger class of graphs, so called tree-like partial Hamming graphs. We investigate these graphs and show some results which imply previously-known results on tree-like partial cubes. For instance, we characterize tree-like partial Hamming graphs and prove that every tree-like partial Hamming graph G contains a Hamming graph that is invariant under every automorphism of G . The latter result is a direct consequence of the result about the dismantlability of the intersection graph of maximal Hamming graphs of a tree-like partial Hamming graph.

Keywords: partial Hamming graph, expansion procedure, dismantlable graph, gated subgraph, intersection graph.

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