Discussiones Mathematicae
Graph Theory 34 (2014) 137–150
doi:10.7151/dmgt.1723

TREE-LIKE PARTIAL HAMMING GRAPHS

Tanja Gologranc

Institute of Mathematics, Physics and Mechanics
Jadranska 19, 1000 Ljubljana, Slovenia

e-mail: tanja.gologranc@imfm.si

Abstract

Tree-like partial cubes were introduced in [B. Brešar, W. Imrich, S. Klavžar, *Tree-like isometric subgraphs of hypercubes*, Discuss. Math. Graph Theory, 23 (2003), 227–240] as a generalization of median graphs. We present some incorrectnesses from that article. In particular we point to a gap in the proof of the theorem about the dismantlability of the cube graph of a tree-like partial cube and give a new proof of that result, which holds also for a bigger class of graphs, so called tree-like partial Hamming graphs. We investigate these graphs and show some results which imply previously-known results on tree-like partial cubes. For instance, we characterize tree-like partial Hamming graphs and prove that every tree-like partial Hamming graph G contains a Hamming graph that is invariant under every automorphism of G. The latter result is a direct consequence of the result about the dismantlability of the intersection graph of maximal Hamming graphs of a tree-like partial Hamming graph.

Keywords: partial Hamming graph, expansion procedure, dismantlable graph, gated subgraph, intersection graph.

2010 Mathematics Subject Classification: 05C12, 05C75.

References

[1] H.-J. Bandelt, *Retracts of hypercubes*, J. Graph Theory **8** (1984) 501–510. doi:10.1002/jgt.3190080407

[2] H.-J. Bandelt and H.M. Mulder, *Helly Theorems for dismantlable graphs and pseudo-modular graphs*, in: R. Bodendiek, R. Henn (Eds.), Topics in Combinatorics and Graph Theory, Physica-Verlag (1990) 65–71. doi:10.1007/978-3-642-46908-4_7

[3] H.-J. Bandelt, H.M. Mulder and E. Wilkeit, *Quasi-median graphs and algebras*, J. Graph Theory **18** (1994) 681–703. doi:10.1002/jgt.3190180705

[4] H.-J. Bandelt and M. van de Vel, *A fixed cube theorem for median graphs*, Discrete Math. **62** (1987) 129–137.
doi:10.1016/0012-365X(87)90022-7

[5] H.-J. Bandelt and M. van de Vel, *Superextensions and the depth of median graphs*, J. Combin. Theory (A) **57** (1991) 187–202.
doi:10.1016/0097-3165(91)90044-H

[6] B. Brešar, *Arboreal structure and regular graphs of median-like classes*, Discuss. Math. Graph Theory **23** (2003) 215–225.
doi:10.7151/dmgt.1198

[7] B. Brešar, *Partial Hamming graphs and expansion procedures*, Discrete Math. **237** (2001) 13–27.
doi:10.1016/S0012-365X(00)00362-9

[8] B. Brešar, W. Imrich and S. Klavžar, *Tree-like isometric subgraphs of hypercubes*, Discuss. Math. Graph Theory **23** (2003) 227–240.
doi:10.7151/dmgt.1199

[9] V. Chepoi, *Bridged graphs are cop-win graphs: an algorithmic proof*, J. Combin. Theory (B) **69** (1997) 97–100.
doi:10.1006/jctb.1996.1726

[10] V. Chepoi, *Isometric subgraphs of Hamming graphs and d-convexity*, Cybernetics **1** (1988) 6–9.
doi:10.1007/BF01069520

[11] F.R.K. Chung, R.L. Graham and M.E. Saks, *A dynamic location problem for graphs*, Combinatorica **9** (1989) 111–131.
doi:10.1007/BF02124674

[12] R.L. Graham and H.O. Pollak, *On addressing problem for loop switching*, Bell System Tech. J. **50** (1971) 2495–2519.
doi:10.1002/j.1538-7305.1971.tb02618.x

[13] R.L. Graham and P. Winkler, *On isometric embeddings of graphs*, Trans. Amer. Math. Soc. **288** (1985) 527–539.
doi:10.1090/S0002-9947-1985-0776391-5

[14] W. Imrich and S. Klavžar, *A convexity lemma and expansion procedures for bipartite graphs*, European J. Combin. **19** (1998) 677–685.
doi:10.1006/eujc.1998.0229

[15] W. Imrich and S. Klavžar, Product Graphs: Structure and Recognition (John Wiley & Sons, New York, 2000).

[16] S. Klavžar and H.M. Mulder, *Median graphs: characterizations, location theory and related structures*, J. Combin. Math. Combin. Comput. **30** (1999) 103–127.

[17] H.M. Mulder, *The structure of median graphs*, Discrete Math. **24** (1978) 197–204.
doi:10.1016/0012-365X(78)90199-1

[18] H.M. Mulder, *n-cubes and median graphs*, J. Graph Theory **4** (1980) 107–110. doi:10.1002/jgt.3190040112

[19] H.M. Mulder, The Interval Function of a Graph (Mathematical Centre Tracts 132, Mathematisch Centrum, Amsterdam, 1980).

[20] H.M. Mulder, *The expansion procedure for graphs*, in: R. Bodendiek (Ed.), Contemporary Methods in Graph Theory (Wissenschaftsverlag, Mannheim, 1990) 459–477.

[21] R. Nowakowski and P. Winkler, *Vertex-to-vertex pursuit in a graph*, Discrete Math. **43** (1983) 235–239.
doi:10.1016/0012-365X(83)90160-7

[22] C. Tardif, *Prefibers and the Cartesian product of metric spaces*, Discrete Math. **109** (1992) 283–288.
doi:10.1016/0012-365X(92)90298-T

[23] E. Wilkeit, *Isometric embeddings in Hamming graphs*, J. Combin. Theory (B) **50** (1990) 179–197.
doi:10.1016/0095-8956(90)90073-9

[24] E. Wilkeit, *The retracts of Hamming graphs*, Discrete Math. **102** (1992) 197–218.
doi:10.1016/0012-365X(92)90054-J

[25] P. Winkler, *Isometric embeddings in products of complete graphs*, Discrete Appl. Math. **7** (1984) 221–225.
doi:10.1016/0166-218X(84)90069-6

Received 16 August 2012
Revised 16 January 2013
Accepted 17 January 2013