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# MAXIMUM HYPERGRAPHS WITHOUT REGULAR SUBGRAPHS

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**Abstract**

We show that an $n$-vertex hypergraph with no $r$-regular subgraphs has at most $2^{n-1}+r-2$ edges. We conjecture that if $n > r$, then every $n$-vertex hypergraph with no $r$-regular subgraphs having the maximum number of edges contains a full star, that is, $2^{n-1}$ distinct edges containing a given vertex. We prove this conjecture for $n \geq 425$. The condition that $n > r$ cannot be weakened.

**Keywords:** hypergraphs, set system, subgraph, regular graph.

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## References

[1] N. Alon, S. Friedland, and G. Kalai, *Regular subgraphs of almost regular graphs*, J. Combin. Theory (B) **37** (1984) 79–91.
doi:10.1016/0095-8956(84)90047-9

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[2] M. Kano, *Regular subgraphs of a regular graph*, Annals of the New York Academy of Sciences **576** (1989) 281–284.
doi:10.1111/j.1749-6632.1989.tb16409.x

[3] D. Mubayi and J. Verstraëte, *Two-regular subgraphs of hypergraphs*, J. Combin. Theory (B) **99** (2009) 643–655.
doi:10.1016/j.jctb.2008.10.005

[4] L. Pyber, *Regular subgraphs of dense graphs*, Combinatorica **5** (1985) 347–349.
doi:10.1007/BF02579250

[5] L. Pyber, V. Rödl, and E. Szemerédi, *Dense graphs without* 3*-regular subgraphs*, J. Combin. Theory (B) **63** (1995) 41–54.
doi:10.1006/jctb.1995.1004

[6] V. Rödl and B. Wysocka, *Note on regular subgraphs*, J. Graph Theory **24** (1997) 139–154.
doi:10.1002/(SICI)1097-0118(199702)24:2⟨139::AID-JGT2⟩3.0.CO;2-R

[7] V.A. Tashkinov, 3*-regular subgraphs of* 4*-regular graphs*, Mat. Zametki **36** (1984) 239–259..

[8] V.A. Tashkinov, *Regular parts of regular pseudographs*, Mat. Zametki **43** (1988)) 263–275.

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