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ON THE DETERMINANT OF q-DISTANCE MATRIX OF A GRAPH

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Abstract

In this note, we show how the determinant of the q-distance matrix $D_q(T)$ of a weighted directed graph G can be expressed in terms of the corresponding determinants for the blocks of G, and thus generalize the results obtained by Graham *et al.* [R.L. Graham, A.J. Hoffman and H. Hosoya, *On the distance matrix of a directed graph*, J. Graph Theory **1** (1977) 85–88]. Further, by means of the result, we determine the determinant of the q-distance matrix of the graph obtained from a connected weighted graph G by adding the weighted branches to G, and so generalize in part the results obtained by Bapat *et al.* [R.B. Bapat, S. Kirkland and M. Neumann, *On distance matrices and Laplacians*, Linear Algebra Appl. **401** (2005) 193–209]. In particular, as a consequence, determinantal formulae of q-distance matrices for unicyclic graphs and one class of bicyclic graphs are presented.

Keywords: q-distance matrix, determinant, weighted graph, directed graph.

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