Discussiones Mathematicae
Graph Theory 34 (2014) 95–102
doi:10.7151/dmgt.1718

THE MINIMUM SPECTRAL RADIUS OF SIGNLESS LAPLACIAN OF GRAPHS WITH A GIVEN CLIQUE NUMBER

Li Su[1], Hong-Hai Li[2] and Jing Zhang

College of Mathematics and Information Science
Jiangxi Normal University
Nanchang, 330022, P.R. China

e-mail: suli@jxnu.edu.cn
lhh@mail.ustc.edu.cn
zhangjing3611@163.com

Abstract

In this paper we observe that the minimal signless Laplacian spectral radius is obtained uniquely at the kite graph $PK_{n-\omega,\omega}$ among all connected graphs with n vertices and clique number ω. In addition, we show that the spectral radius μ of $PK_{m,\omega}$ $(m \geq 1)$ satisfies

$$\frac{1}{2}(2\omega - 1 + \sqrt{4\omega^2 - 12\omega + 17}) \leq \mu \leq 2\omega - 1.$$

More precisely, for $m > 1$, μ satisfies the equation

$$\mu - \omega - \frac{\omega - 1}{\mu - 2\omega + 3} = a_m\sqrt{\mu^2 - 4\mu} + \frac{1}{t_1},$$

where $a_m = \frac{1}{1-t_1^{2m+3}}$ and $t_1 = \frac{\mu-2+\sqrt{(\mu-2)^2-4}}{2}$. At last the spectral radius $\mu(PK_{\infty,\omega})$ of the infinite graph $PK_{\infty,\omega}$ is also discussed.

Keywords: clique number, kite graph, signless Laplacian, spectral radius.
2010 Mathematics Subject Classification: 05C50,15A18.

[1]Supported by the Provincial Natural Science Foundation of Jiangxi (No.2010GZS0093), the Young Growth Foundation of Jiangxi Normal University (No. 4555).
[2]Supported by National Natural Science Foundation of China (No.11201198 and No. 11026143), Natural Science Foundation of Jiangxi Province (No.20132BAB201013), the Sponsored Program for Cultivating Youths of Outstanding Ability in Jiangxi Normal University. Corresponding author.

References

[1] Y. Chen, *Properties of spectra of graphs and line graphs*, Appl. Math. J. Chinese Univ. (B) **17** (2002) 371–376.
doi:10.1007/s11766-002-0017-7

[2] D. Cvetković, P. Rowlinson and S.K. Simić, *Signless Laplacians of finite graphs*, Linear Algebra Appl. **423** (2007) 155–171.
doi:10.1016/j.laa.2007.01.009

[3] D. Cvetković and S.K. Simić, *Towards a spectral theory of graphs based on signless Laplacian* I, Publ. Inst. Math. (Beograd) **99** (2009) 19–33.

[4] D. Cvetković and S.K. Simić, *Towards a spectral theory of graphs based on signless Laplacian* II, Linear Algebra Appl. **432** (2010) 2257–2272.
doi:10.1016/j.laa.2009.05.020

[5] E.R. van Dam and W. Haemers, *Which graphs are determined by their spectrum?*, Linear Algebra Appl. **373** (2003) 241–272.
doi:10.1016/S0024-3795(03)00483-X

[6] W. Haemers and E. Spence, *Enumeration of cospectral graphs*, European J. Combin. **25** (2004) 199–211.
doi:10.1016/S0195-6698(03)00100-8

[7] B. Mohar and W. Woess, *A survey on spectra of infnite graphs*, Bull. London Math. Soc. **21** (1989) 209–234.
doi:10.1112/blms/21.3.209

[8] B. Mohar, *On the Laplacian coefficients of acyclic graphs*, Linear Algebra Appl. **722** (2007) 736–741.
doi:10.1016/j.laa.2006.12.005

[9] D. Stevanović and P. Hansen, *The minimum spectral radius of graphs with a given clique number*, Electron. J. Linear Algebra **17** (2008) 110–117.

Received 15 March 2012
Revised 10 November 2012
Accepted 13 December 2012