Note

STAR-CYCLE FACTORS OF GRAPHS

Yoshimi Egawa
Tokyo University of Science, Shinjuku-Ku, Tokyo, Japan

Mikio Kano\(^1\)
Ibaraki University, Hitachi, Ibaraki, Japan
e-mail: kano@max.ibaraki.ac.jp

AND

Zheng Yan
Ibaraki University, Hitachi, Ibaraki, Japan
e-mail: yanzhenghubei@163.com

Abstract

A spanning subgraph \(F\) of a graph \(G\) is called a star-cycle factor of \(G\) if each component of \(F\) is a star or cycle. Let \(G\) be a graph and \(f: V(G) \to \{1,2,3,\ldots\}\) be a function. Let \(W = \{v \in V(G) : f(v) = 1\}\). Under this notation, it was proved by Berge and Las Vergnas that \(G\) has a star-cycle factor \(F\) with the property that (i) if a component \(D\) of \(F\) is a star with center \(v\), then \(\deg_F(v) \leq f(v)\), and (ii) if a component \(D\) of \(F\) is a cycle, then \(V(D) \subseteq W\) if and only if \(\text{iso}(G - S) \leq \sum_{x \in S} f(x)\) for all \(S \subset V(G)\), where \(\text{iso}(G - S)\) denotes the number of isolated vertices of \(G - S\). They proved this result by using circulation theory of flows and fractional factors of graphs. In this paper, we give an elementary and short proof of this theorem.

Keywords: star factor, cycle factor, star-cycle factor, factor of graph.

2010 Mathematics Subject Classification: 05C70.

References

\(^1\)The author was supported by Japan Society for the Promotion of Science, Grant-in-Aid for Scientific Research (C).


Received 28 August 2012
Revised 13 December 2012
Accepted 28 December 2012