STAR-CYCLE FACTORS OF GRAPHS

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Abstract

A spanning subgraph $F$ of a graph $G$ is called a star-cycle factor of $G$ if each component of $F$ is a star or cycle. Let $G$ be a graph and $f : V(G) \to \{1, 2, 3, \ldots\}$ be a function. Let $W = \{v \in V(G) : f(v) = 1\}$. Under this notation, it was proved by Berge and Las Vergnas that $G$ has a star-cycle factor $F$ with the property that (i) if a component $D$ of $F$ is a star with center $v$, then $\deg_F(v) \leq f(v)$, and (ii) if a component $D$ of $F$ is a cycle, then $V(D) \subseteq W$ if and only if $\text{iso}(G - S) \leq \sum_{x \in S} f(x)$ for all $S \subseteq V(G)$, where $\text{iso}(G - S)$ denotes the number of isolated vertices of $G - S$. They proved this result by using circulation theory of flows and fractional factors of graphs. In this paper, we give an elementary and short proof of this theorem.

Keywords: star factor, cycle factor, star-cycle factor, factor of graph.

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