## Note

# STAR-CYCLE FACTORS OF GRAPHS 

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#### Abstract

A spanning subgraph $F$ of a graph $G$ is called a star-cycle factor of $G$ if each component of $F$ is a star or cycle. Let $G$ be a graph and $f: V(G) \rightarrow$ $\{1,2,3, \ldots\}$ be a function. Let $W=\{v \in V(G): f(v)=1\}$. Under this notation, it was proved by Berge and Las Vergnas that $G$ has a star-cycle factor $F$ with the property that (i) if a component $D$ of $F$ is a star with center $v$, then $\operatorname{deg}_{F}(v) \leq f(v)$, and (ii) if a component $D$ of $F$ is a cycle, then $V(D) \subseteq W$ if and only if $\operatorname{iso}(G-S) \leq \sum_{x \in S} f(x)$ for all $S \subset V(G)$, where $\operatorname{iso}(G-S)$ denotes the number of isolated vertices of $G-S$. They proved this result by using circulation theory of flows and fractional factors of graphs. In this paper, we give an elementary and short proof of this theorem.


Keywords: star factor, cycle factor, star-cycle factor, factor of graph.
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