

A NOTE ON THE PERMANENTAL ROOTS OF BIPARTITE GRAPHS¹

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Abstract

It is well-known that any graph has all real eigenvalues and a graph is bipartite if and only if its spectrum is symmetric with respect to the origin. We are interested in finding whether the permanental roots of a bipartite graph G have symmetric property as the spectrum of G . In this note, we show that the permanental roots of bipartite graphs are symmetric with respect to the real and imaginary axes. Furthermore, we prove that any graph has no negative real permanental root, and any graph containing at least one edge has complex permanental roots.

Keywords: permanent, permanental polynomial, permanental roots.

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